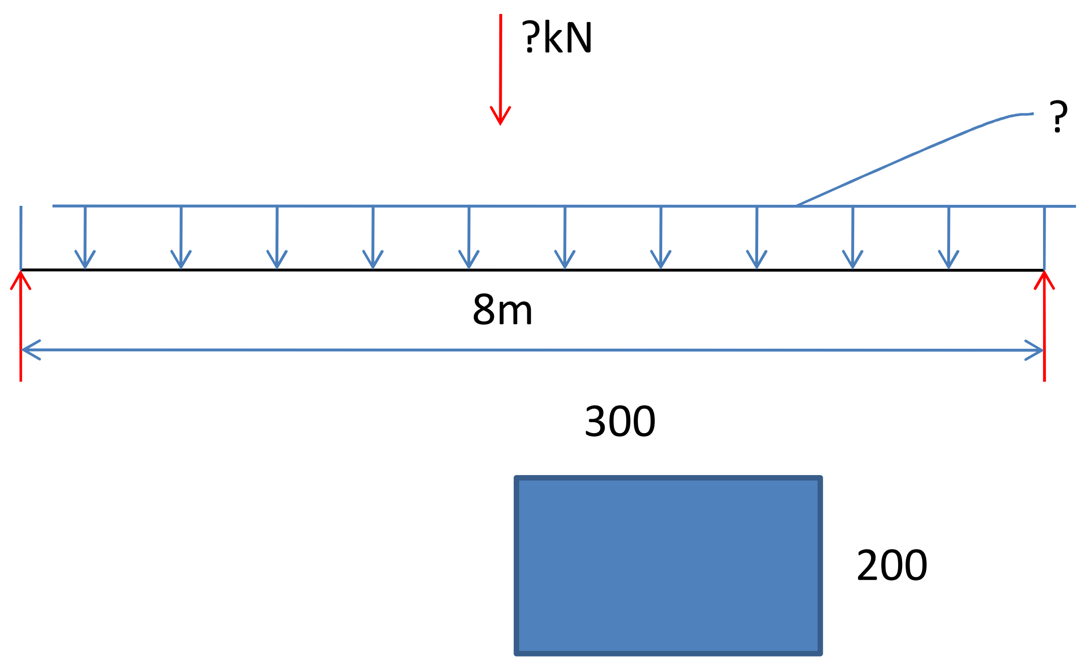
- A rectangular beam 200 mm deep and 300 mm wide is simply supported over a span of 8 m. What uniformly distributed load per metre the beam may carry, if the bending stress is not to exceed $120 N/mm^2$. Also find max. Central concentrated load it can carry.



$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$M = \sigma_{\max} \frac{I}{y_{\max}} = \sigma_{\max} Z$$

$$I = \frac{bd^3}{12}; y_{\max.} = \frac{d}{2};$$

$Z=2x10^6$

$$z = \frac{I}{y_{\max.}} = \frac{bd^2}{6}$$

For a S.S. Beam with u.d.l, max.

$$BM = \frac{wL^2}{8} = \frac{w \times 8^2}{8} = 8w \quad Nm = 8000w \quad Nmm$$

$$BM = \sigma_{\max} . Z = 8000\ w$$

$$120 \times 2000000 = 8000\ w$$

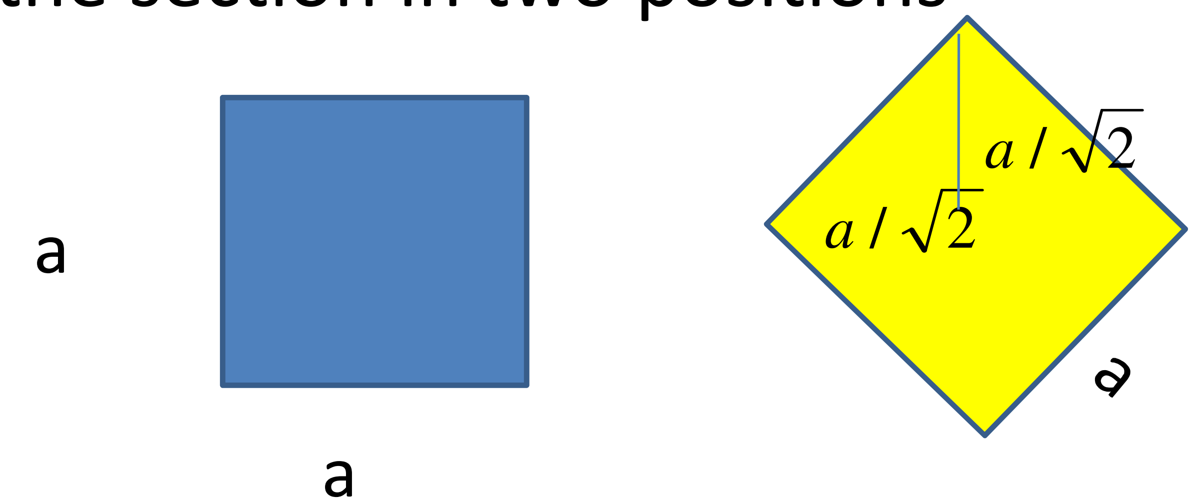
$$w = 30{,}000\ N/m = 30\ kN/m$$

$$120 \times 2000000 = \frac{WL}{4} = \frac{W \times 8}{4}$$

$$= 2W \quad Nm = 2000\ W \quad Nmm$$

$$W = 120\ kN$$

- A beam is of square section of side ‘a’. bending stress is σ
- Find the moment of resistance when the beam section is placed such that(i) two sides are horizontal,(ii) one diagonal is vertical. Find also the ratio of the moment of resistance of the section in two positions



$$I = \frac{bd^3}{12};\, y_{max.} = \frac{d}{2};$$

$$z = \frac{I}{y_{max.}} = \frac{bd^2}{6} = \frac{a^3}{6}$$

$$M_1 = \sigma\frac{a^3}{6}$$

$$I = 2\times\frac{bh^3}{12};\, y_{max.} = \frac{a}{\sqrt{2}};$$

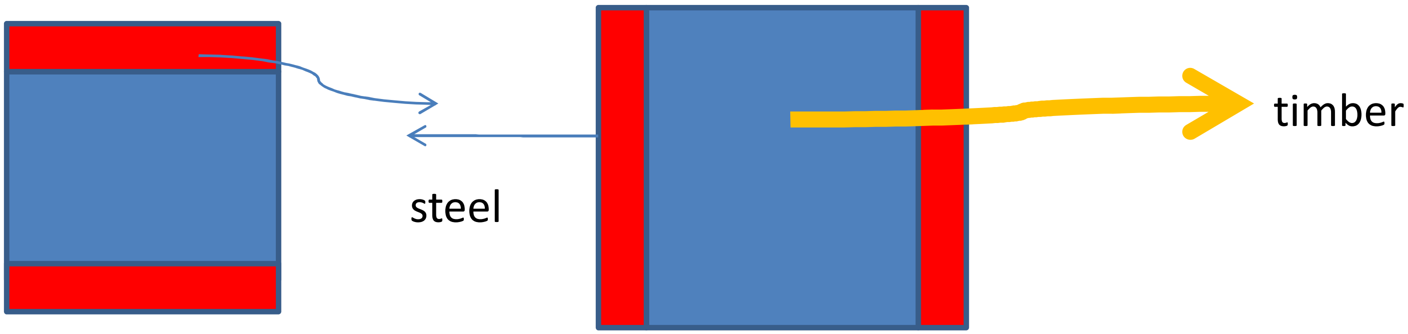
$$z = \frac{I}{y_{max.}} = \frac{2\times\frac{\sqrt{2}a(\frac{a}{\sqrt{2}})^3}{12}}{\frac{a}{\sqrt{2}}} = \frac{a^3}{6\sqrt{2}}$$

$$M_2 = \sigma\frac{a^3}{6\sqrt{2}}$$

$$\frac{M_1}{M_2} = \sqrt{2}$$

Composite beam(Flitched beam)

- Consists of two different materials---eg. Timber beam reinforced with steel plates at top and bottom or on sides



- 2 materials are connected rigidly, strain same at any distance from neutral axis

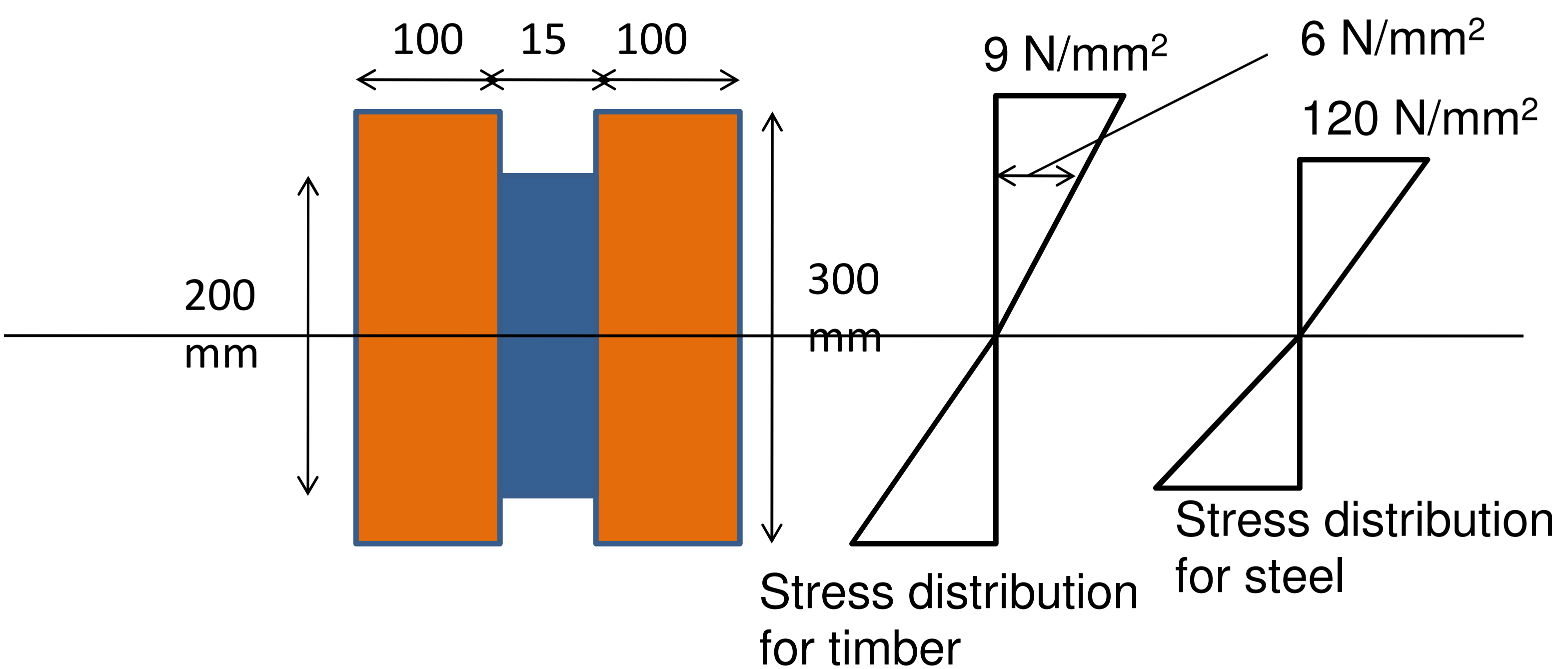
$$\frac{\sigma_1}{E1} = \frac{\sigma_2}{E_2}$$

Total resisting moment=sum of resisting moment caused by individual material

M=M1+M2

A composite beam consist of two timber joists 100 mm wide and 300 mm deep with a steel plate 200 mm deep and 15 mm thick placed symmetrically in between and clamped to them. Calculate the total moment of resistance of the section if the allowable stress in the joist is 9 N/mm² assume $E_s = 20E_w$

A.



In the stress diagram for timber the allowable stress is 9 N/mm^2 (maximum in joist)

at the level of steel, stress in timber $= \frac{9}{150} \times 100 = 6 N/mm^2$

We know that $\frac{\sigma_s}{E_s} = \frac{\sigma_w}{E_w}$

$$\therefore \sigma_s = \sigma_w \frac{E_s}{E_w} = 6 \times 20 = 120 N/mm^2$$

This is the maximum stress in steel

$$M = M_S + M_W = \sigma_s Z_s + \sigma_w Z_w$$

$$Z_s = \frac{bd^2}{6} = \frac{15 \times 200^2}{6} = 10^5 mm^3$$

$$Z_s = \frac{bd^2}{6} = \frac{200 \times 300^2}{6} = 3 \times 10^6 mm^3$$

$$M = M_S + M_W = \sigma_s Z_s + \sigma_w Z_w$$

$$= 120 \times 10^5 + 9 \times 3 \times 10^6 = 39 \times 10^6 Nmm$$

BEAM OF UNIFORM STRENGTH

- The beam considered so far has uniform cross section throughout the span.

- Stresses in the extreme fibres of the section vary from section to section along the span.

- All extreme fibres are not loaded to their maximum capacity.

- A simply supported beam with symmetrical loading, BM_{max} occurs at midspan.
- Maximum bending stress corresponding to maximum BM
- But towards the support **BM** reduces to zero
- This implies that there is wastage of material in the cross section of the beam.

- ✓ A beam is designed in such that every extreme fibre along the span is loaded to its maximum permissible stress by varying the cross section, then it is known as **beam of uniform strength**

- ✓ For extreme fibre stress to be kept same at every section, bending moment at any section should be proportional to section modulus.

- ✓ Since $M = \sigma . Z \, ; M \, \alpha Z$ (σ is kept constant)

- ✓ The cross section of the ideal beam can be obtained by the following ways

beam of uniform strength

i. Keeping the width constant throughout and varying the depth.

ii. Keeping the depth constant throughout the span and varying the width

iii. By varying both width and depth

Eg. The leaf spring or laminated spring used as shock absorbers in heavy vehicles.

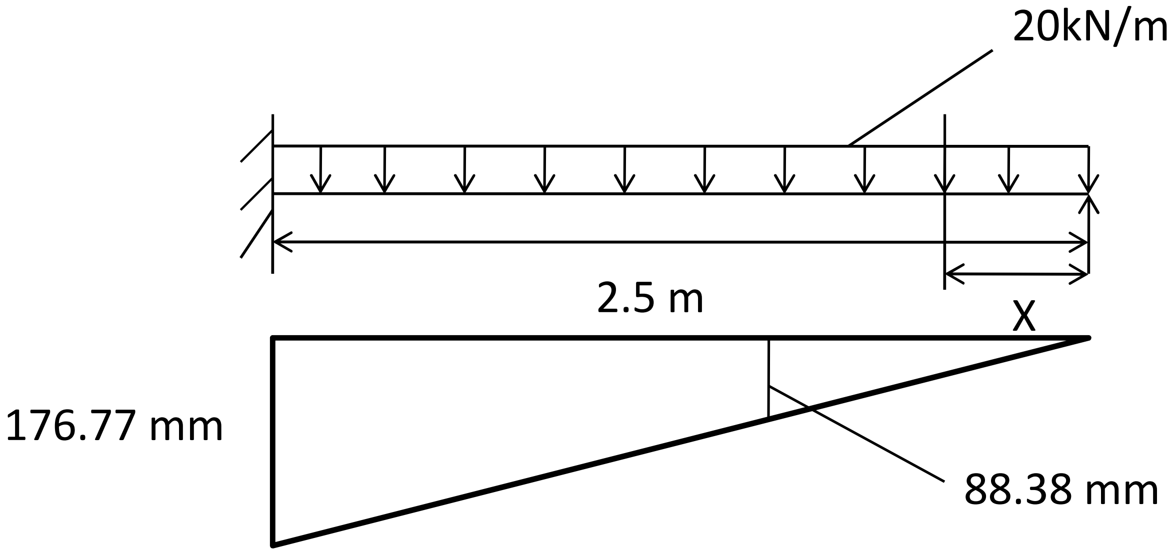
Q. A cantilever, 2.5 m long carries a UDL of 20 kN/m. The breadth of the section remains constant and is equal to 100mm. Determine the depth of the section at midspan of the cantilever and at fixed end if stress remains same throughout as 120 N/mm^2

A.

Bending moment at x distance from free end,

$$M = \frac{wx^2}{2} = \sigma . Z$$

$$w = 20\, kN / m = \frac{20 \times 10^3}{10^3} = 20\, N / mm$$



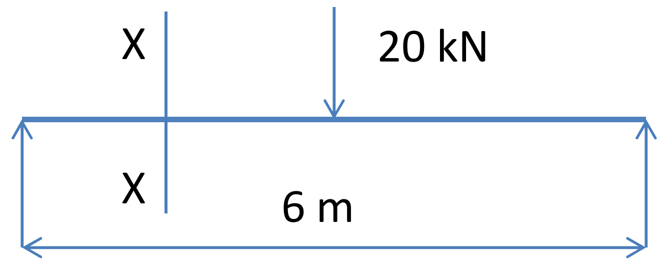
$$\frac{20x^2}{2} = 120 \times \frac{bd^2}{6} \rightarrow 10x^2 = \frac{120 \times 100 \times d^2}{6}$$

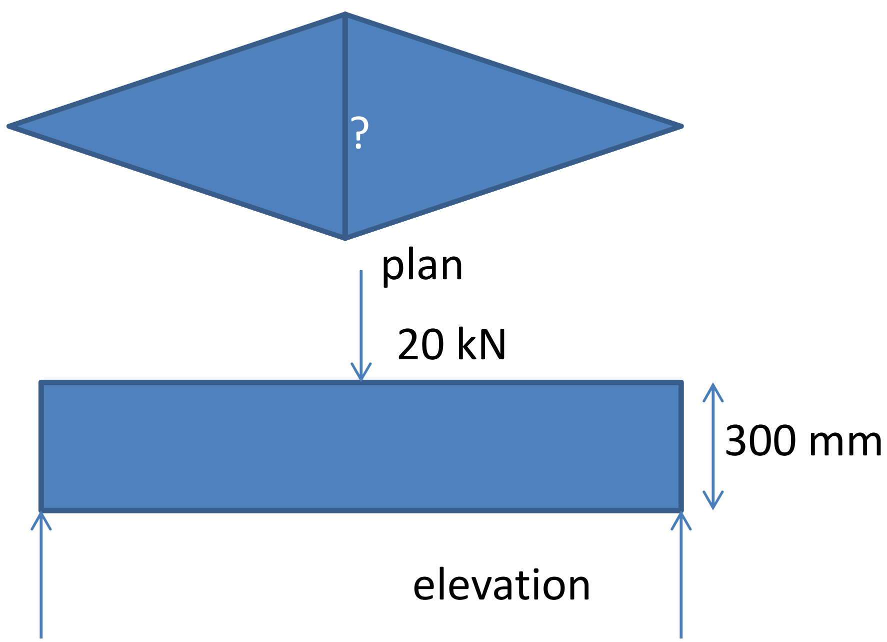
$$d = \sqrt{\frac{60x^2}{12000}} = x.\sqrt{\frac{1}{200}}$$

At x =l/2; d = 88.38mm& At x=l =2500; d = 88.38mm

Q.Determine the cross section of a rectangular beam of uniform strength for a simply supported beam of 6m span subjected to a central concentrated load of 20kN by keeping a depth of 300mm throughout .take permissible strength as $8N/mm^2$

Ans: Bending moment at a section x distance from left support,


X
20 kN
X
6 m


?
plan
20 kN
300 mm
elevation

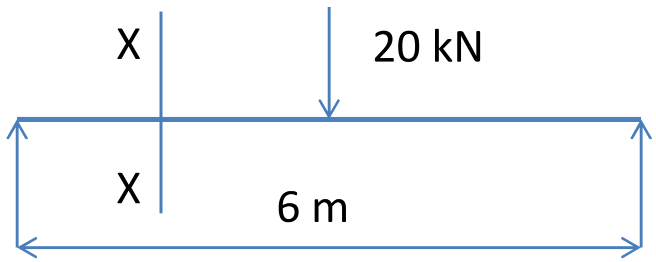
$$M = \frac{W}{2}\, x \text{ x } 10^3 \text{x } 10^3 = \sigma \text{ x Z}$$

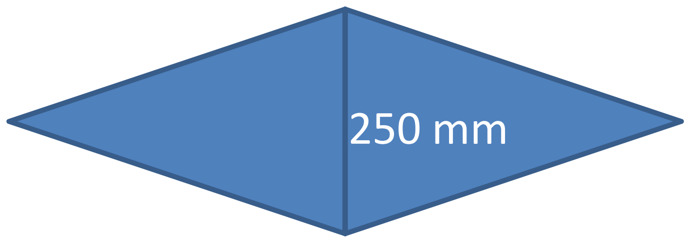
$$\frac{W}{2}\, x \text{ x } 10^3 \text{x } 10^3 = 8 \text{ x} \frac{bd^2}{6} = 8 \text{ x } b \text{ x} \frac{300^2}{6}$$

$$\text{b} = \frac{Wx \text{ x } 10^6 \text{x } 6}{2 \text{ x } 8 \text{ x } 300^2} = 4.166Wx = 83.33x \text{ mm}$$

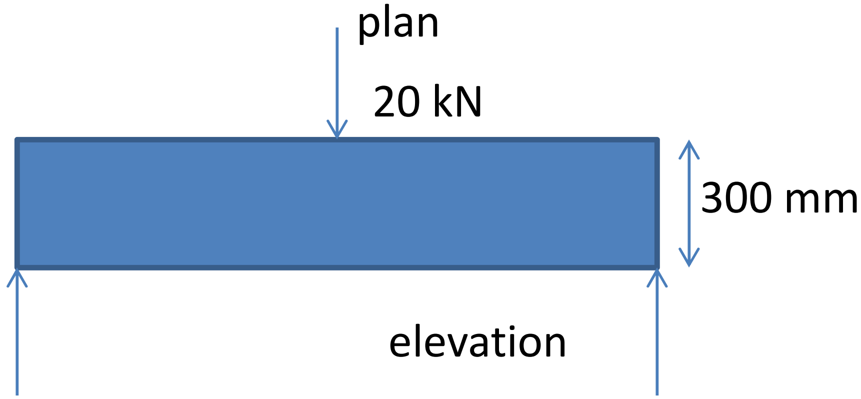
At x = 0, b = 0

At x = l/2, b = 83.33 x 3 = 250 mm





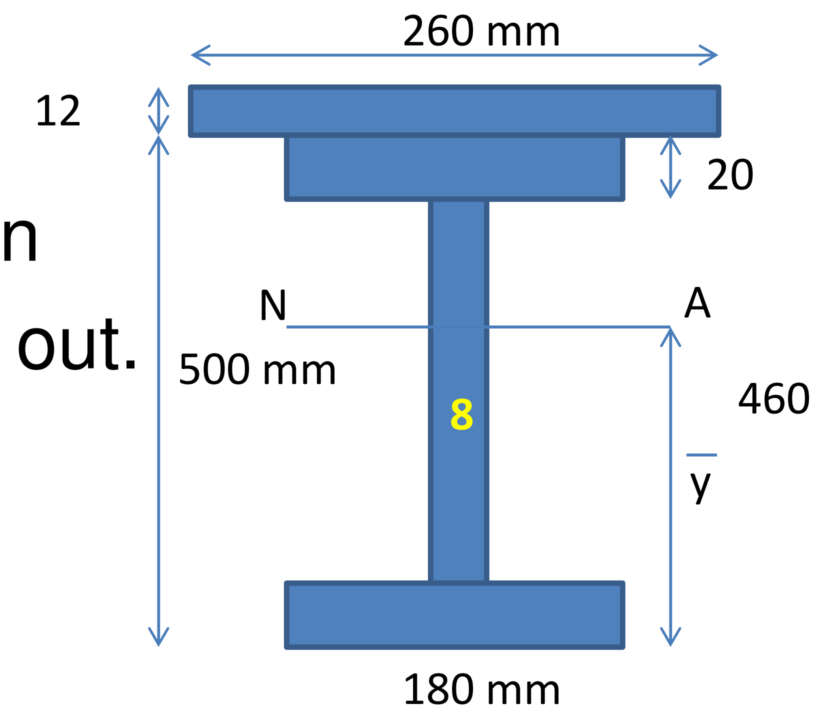
plan



elevation

Q. A symmetric I section has flanges of size 180 mm x 20 mm and its overall depth 500 mm. thickness of web is 8 mm. it is strengthened with a plate of size 260 mm x 12 mm on compression side. Find the moment of resistance of section if permissible stress is 170 N/mm^2. How much UDL it can carry if it is used as a cantilever beam of span 3 m?

A: Since, the section is unsymmetrical, first the position of neutral axis has to be found out.



Portion	Area, A mm^2	y, mm	Ay
Bottom flange	180 x 20	10	36000
Web	460 x 8	250	920000
Top flange	180 x 20	490	1764000
Top plate	260 x 12	506	1578720

$$y = \frac{\Sigma Ay}{\Sigma A} = \frac{4298720}{14000} = 307.5\ mm$$

Permissible stress, $\sigma = 170 \frac{N}{mm^2}$

$$I_{XX} = \frac{180 \text{ x } 20^3}{12} + 180 \text{ x } 20(307.5 - 10)^2 + \frac{8 \text{ x } 460^3}{12}$$

$$+ 8 \text{ x } 460(250 - 307.5)^2 + \frac{180 \text{ x } 20^3}{12}$$

$$+ 180 \text{ x } 20(490 - 307.5)^2 + \frac{260 \text{ x } 12^3}{12}$$

$$+ 260 \text{ x } 12(506 - 307.5)^2 = 6.39 \text{ x } 10^8 mm^4$$

$$M = \sigma \text{ x} \frac{\text{I}}{y_{max}} = 170 \text{ x} \frac{6.39 \text{ x } 10^8}{307.5}$$

$$= 3.5367 \text{ x } 10^8 Nmm$$

Max. BM for a simply supported beam carrying UDL = $\frac{wl^2}{8}$; l=3m

M = 3.536 x 102 kNm = $\frac{wl^2}{8}$

w = 314.37 kN/m