EST 100 Engineering Mechanics

Course Objectives

Goal of this course is to expose the students

- Fundamental concepts of mechanics and enhance their problem-solving skills.
- Understand the influence of applied force system and the geometrical properties of the rigid bodies while stationary or in motion

Course Outcome

At the end of the course, the student should be able to:

- Apply fundamental theorems of mechanics, knowledge about types of force systems and concept of free body and equilibrium to arrive solutions for rigid body with concurrent forces.
- Analyse single bodies and connected bodies using Coulomb's law of friction. Also solve beams subjected to different loads
- Examine properties of surfaces/bodies for application in structural analysis and effect of force systems in space.
- Apply D'Alembert's principle to obtain various forces acting on a moving body.
- Apply the concept of mechanical vibrations and simple harmonic motion to obtain solution for real life dynamic problems involving vibrations and periodic motion.

Module 1

Introduction to Engineering Mechanics-statics-basic principles of statics-Parallelogram law, equilibrium law, principles of superposition and transmissibility, law of action and reaction(review) free body diagrams.

Concurrent coplanar forces-composition and resolution of forces-resultant and equilibrium equations – methods of projections – methods of moments – Varignon's Theorem of moments.

Module 2

Friction – sliding friction - Coulomb's laws of friction – analysis of single bodies –wedges, ladder- analysis of connected bodies .

Parallel coplanar forces – couple - resultant of parallel forces – centre of parallel forces – equilibrium of parallel forces – Simple beam subject to concentrated vertical loads. General coplanar force system - resultant and equilibrium equations.

Module 3

Centroid of composite areas- – moment of inertia-parallel axis and perpendicular axis theorems. Polar moment of inertia,radius of gyration,mass moment of inertia-ring,cylinder and disc.

Theorem of Pappus Guldinus(demonstration only)

Forces in space - vectorial representation of forces, moments and couples –resultant and equilibrium equations – concurrent forces in space (simple problems only)

Module 4

Dynamics – rectilinear translation - equations of kinematics(review)

kinetics – equation of motion – D'Alembert's principle. – motion on horizontal and inclined surfaces, motion of connected bodies. Impulse momentum equation and work energy equation (concepts only).

Curvilinear translation - equations of kinematics –projectile motion(review), kinetics – equation of motion. Moment of momentum and work energy equation (concepts only).

Module 5

Rotation – kinematics of rotation- equation of motion for a rigid body rotating about a fixed axis – rotation under a constant moment.

Plane motion of rigid body – instantaneous centre of rotation (concept only).

Simple harmonic motion – free vibration –degree of freedom- undamped free vibration of spring mass system-effect of damping(concept only)

- Beer and Johnson, Vector Mechanics for Engineers - Statics and Dynamics, Tata Mc-Graw Hill Publishing Company Limited
- Hibbeler R.C., Engineering Mechanics: Statics and Dynamics. Pearson Prentice Hall
- Benjamin J., Engineering Mechanics, Pentex Book Publishers and Distributors
- Kumar K. L., Engineering Mechanics, Tata Mc-Graw Hill Publishing Company Limited
- Tayal A. K., Engineering Mechanics- Statics and Dynamics, Umesh Publications
- S.S.Bhavikkatti, Engineering Mechanics, New Age International Publishers
- Jaget Babu, Engineering Mechanics, Pearson Prentice Hall
- Merriam J. L. and Kraige L. G., Engineering Mechanics – Vol. I and II, John Wiley.
- Rajasekaran S. and G. Sankarasubramanian, Engineering Mechanics, Vikas Publishing House Private Limited

Assessment methods

Continuous Internal Evaluation Pattern:

- Attendance : 10 marks
- Continuous Assessment Test (2 numbers) : 25 marks
- Assignment/Quiz/Course project : 15 marks (Average of 3 assignments)

Total CIE mark = 50

End Semester Examination Pattern:

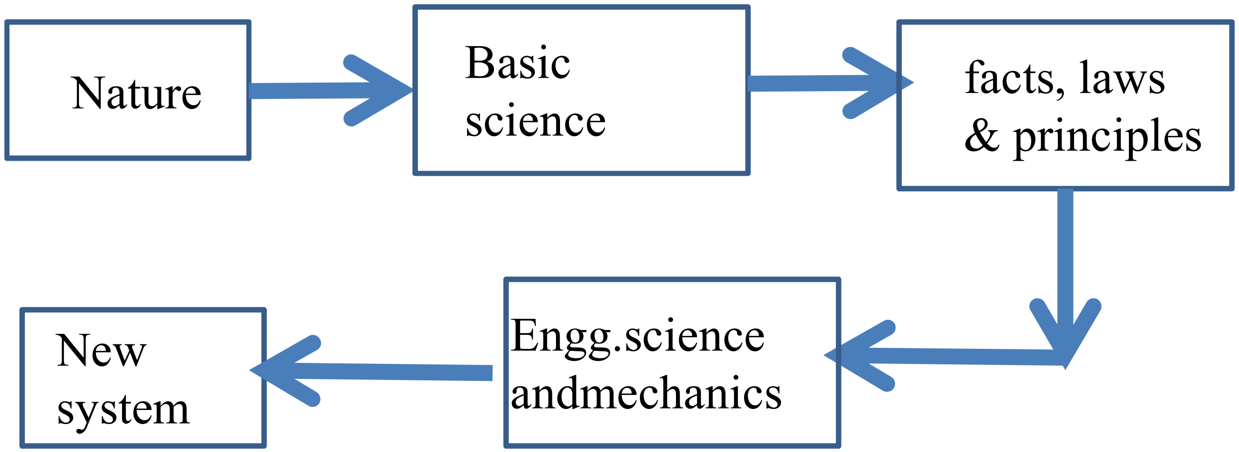
Part A: 10 questions (2 from each module)
carries 3 marks = 30 marks

Part B: 10 questions (2 from each module)
carries 14 marks = 14 x 5 =70 marks

Total ESE marks = 100

Science & Engg.

- Science-understanding & gathering of facts, laws and principles from nature
- Engg. Utilization of facts, laws and principles to create a new phenomena
  - Art of executing partial application of scientific knowledge



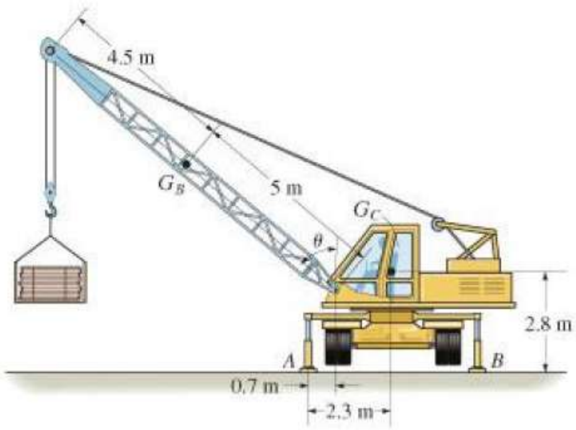
- Mechanics: Oldest of the Physical Sciences
- Archimedes (287-212 BC): Principles of Lever and Buoyancy!
- Mechanics deals with effects of forces on objects
- Branch of engg. that applies principles of mechanics to design the system by considering forces
- Study of behaviour of bodies when they are in rest or motion
- Deals with the state of rest or motion of bodies subjected to the action of forces

Mechanics

1. Statics (action of forces on bodies at rest)
2. Dynamics (motion of bodies under the action of forces)
   - ✓ Kinematics- Study of motion of bodies without reference to the forces (either cause motion or generated due to motion)

     Space time relation of motion of body without considering forces
   - ✓ Kinetics -Study of relationship between forces and resulting motion
3. Hydraulics
   - ✓ Hydrostatics
   - ✓ Hydrodynamics

- **Statics:** deals with equilibrium of bodies under action of forces (bodies may be either at rest or move with a constant velocity).



- **Dynamics:** deals with motion of bodies (accelerated motion)

Mechanics-basic concept

1. Space

2. Time

3. Mass

4. Force

Mechanics-basic concept

1 **Space (Length):** geometric region occupied by the body

- needed to locate position of a point in space, & describe size of the physical system - Distances, Geometric Properties

(linear/angular measurements)

- the position of a point P given in terms of three coordinates measured from a reference point or origin.
- Physical space measured from reference lines/ axes

Basic concept (contd…)

2.Time: measure of successive events-measure of duration of an event and interval between them

Refers to the sequence of events **(by clock)**

3.Mass: quantity of matter in a body

- ✓ measure of inertia of a body (its resistance to change in velocity of motion)
- used to characterize and compare bodies, e.g., response to earth's gravitational attraction and resistance to changes in translational motion.
- $g_{\text{at moon}}$ 1.622 $g_{\text{at earth}}$ 9.81

Mechanics-basic concept

4. Force: the action of one body on another

Eg. May be by actual contact or at a distance (gravitational & magnetic)

characterized by its magnitude, direction of its action, and its point of application

- Force is a Vector quantity
- Force field F= F (x, y, z,t)
- Gravity, push/pull applied to bodies, magnetic, gas pressure, friction
- External and internal force

- **Mechanics: Fundamental Concepts .....**

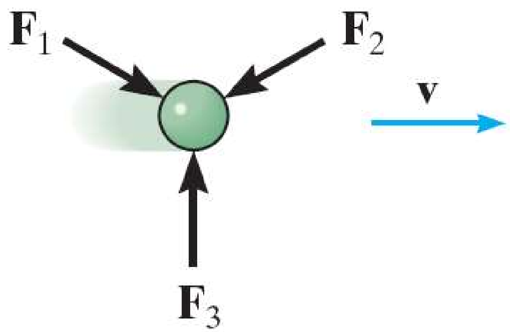
- Mass is a property of matter that does not change from one location to another.
- Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located
- Weight of a body is the gravitational force acting on it.

- In Newtonian Mechanics, space, time, and mass are absolute concepts, independent of each other. Force, however, is not independent of the other three. The force acting on a body is related to the mass of the body and the variation of its velocity with time.

Fundamental Principles

1. ***Newton's First Law*:** If the resultant force on a particle is zero, the particle will remain at rest or continue to move in a straight line.

   First law contains the principle of the equilibrium of forces



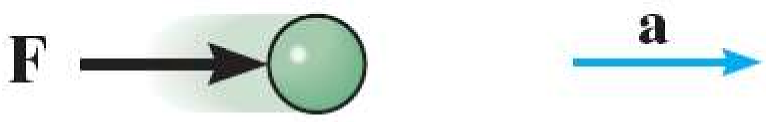
Equilibrium

Fundamental Principles

2. ***Newton's Second Law*:** A particle will have an acceleration proportional to a nonzero resultant applied force.

Second Law forms the basis for most of the analysis in Dynamics

$$\vec{F} = m\vec{a}$$

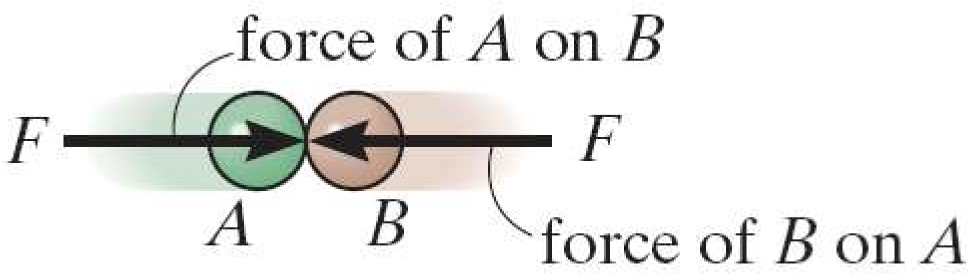


Accelerated motion

Fundamental Principles

3. ***Newton's Third Law*:** The forces of action and reaction between two particles have the same magnitude and line of action with opposite sense.

Forces always occur in pairs of equal and opposite forces.

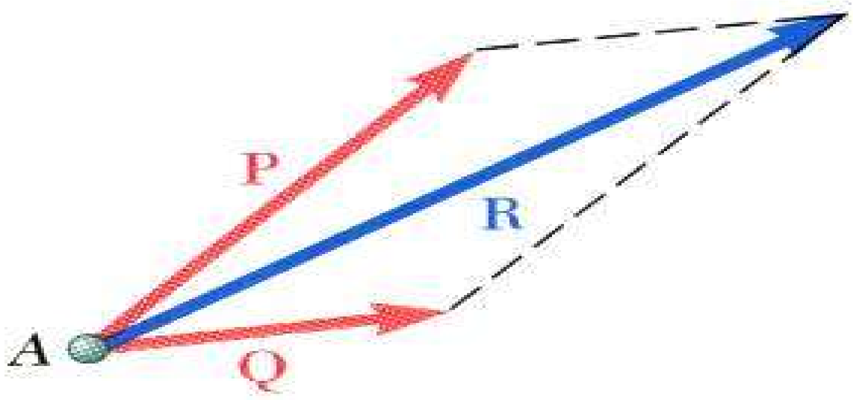


Action – reaction

Fundamental Principles

4.Parallelogram law of forces

- ✓ If two forces acting simultaneously at a point are represented in magnitude and direction by adjacent sides of a parallelogram, then the diagonal of the parallelogram passing through the point of intersection represent the resultant of two forces in both magnitude and direction.

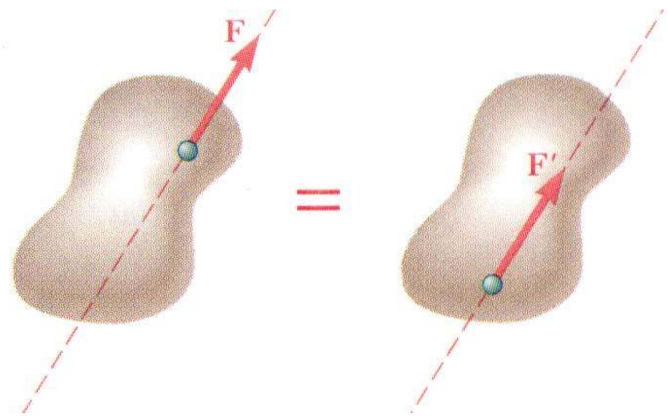


- *Parallelogram Law*

Fundamental Principles

5.Principle of transmissibility

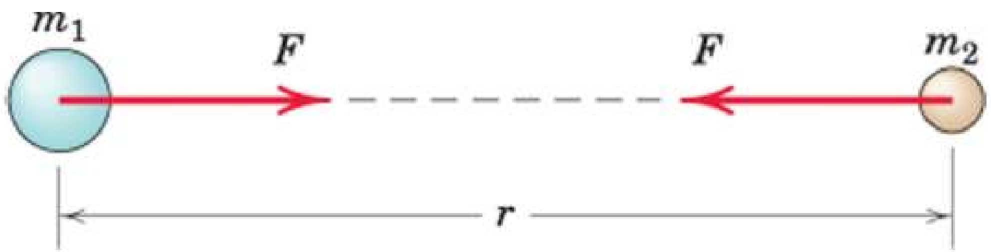
- ✓ The condition of equilibrium/ motion of rigid body will remain unchanged if the point of application of a force acting on a rigid body is transmitted to any other point on the line of action of force



• *Principle of Transmissibility*

6. Newton's Law of Gravitation: Two particles are attracted with equal and opposite forces

$$F = G\frac{m_1 m_2}{r^2}$$



F = mutual force of attraction between two particles

G = universal constant of gravitation

Experiments → G = 6.673x10^{-11} m^3/(kg.s^2)

Rotation of Earth is not taken into account

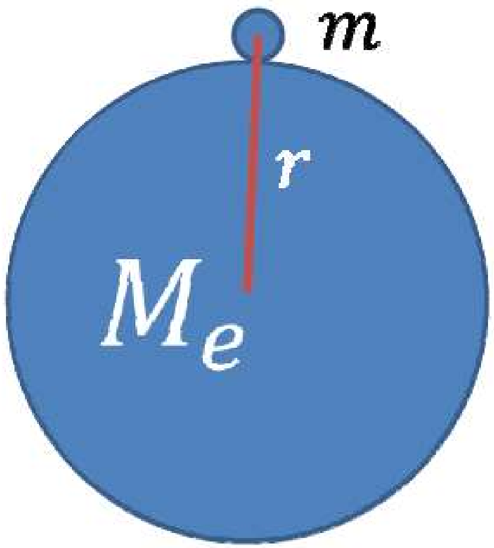
m_1, m_2 = masses of two particles

r = distance between two particles

- Weight of a Body: If a particle is located at or near the surface of the earth, the only significant gravitational force is that between the earth and the particle

Weight of a particle having mass $m_1 = m$:

Assuming earth to be a non-rotating sphere of constant density and having mass $m_2 = M_e$



$$W = G\frac{mM_e}{r^2}$$

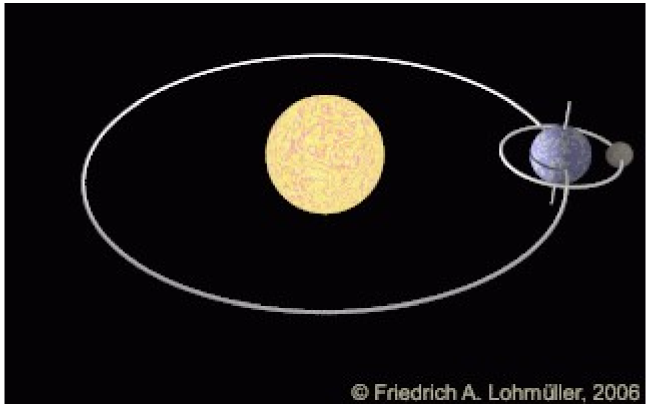
r = distance between the earth's center and the particle

$$W = mg$$

Let $g = G\,M_e/r^2$ = acceleration due to gravity ($9.81 m/s^2$)

- **Mechanics: Idealizations**

- **Particle:** A body with mass but with dimensions that can be neglected
- Size of earth is insignificant compared to the size of its orbit. Earth can be modeled as a particle when studying its orbital motion



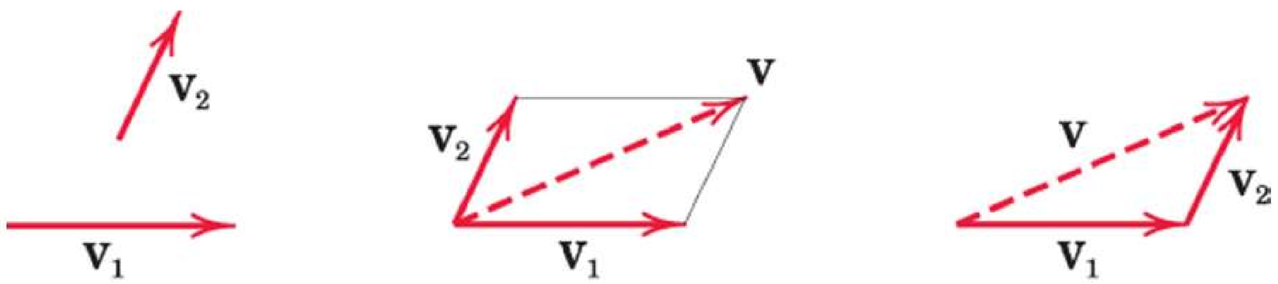
- **Rigid Body**:

- A combination of large number of particles in which all particles remain at a fixed distance (practically) from one another before and after applying a load.
- Material properties of a rigid body are not required to be considered when analyzing the forces acting on the body.
- In most cases, actual deformations occurring in structures, machines, mechanisms, etc. are relatively small, and rigid body assumption is suitable for analysis

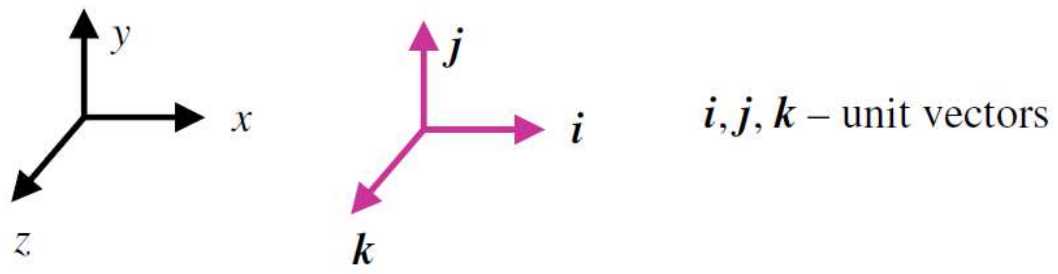
- **Concentrated Force:**
- Effect of a loading which is assumed to act at a point (CG) on a body.
- Provided the area over which the load is applied is very small compared to the overall size of the body.

Ex: Contact Force between a wheel and ground.

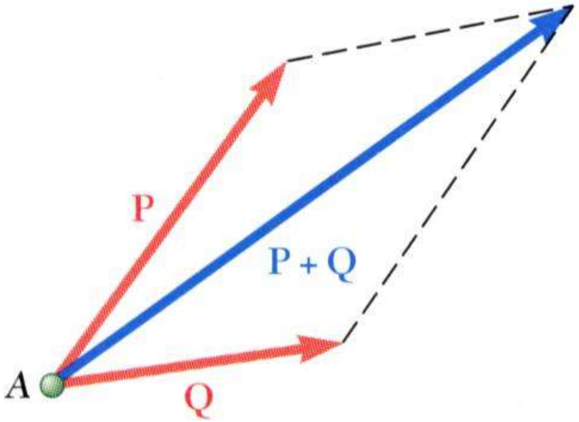
- **Scalars and Vectors**
- Scalars: only magnitude is associated. time, volume, density, speed, energy, mass
- Vectors: possess direction as well as magnitude, and must obey the parallelogram law of addition (and the triangle law). --- displacement, velocity, acceleration, force, moment, momentum
- Equivalent Vector: V = V1 + V2 (Vector Sum)

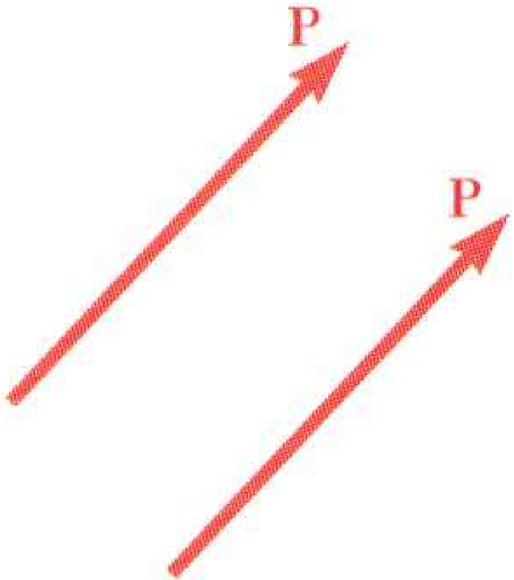


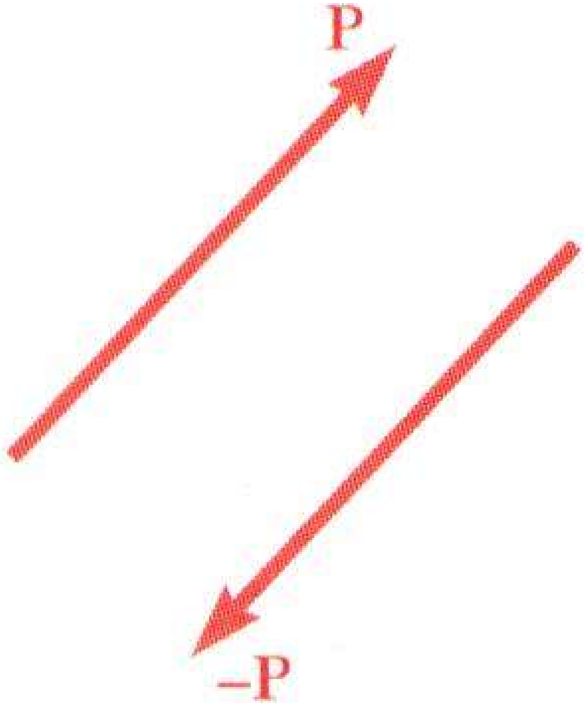
- A Vector V can be written as: **V** = *V*n
- *V* = magnitude of **V**
- n = unit vector whose magnitude is one and whose direction coincides with that of **V**
- Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude
- Vectors represented by Bold and Non-Italic letters (**V**)
- Magnitude of vectors represented by Non-Bold, Italic letters (*V*)



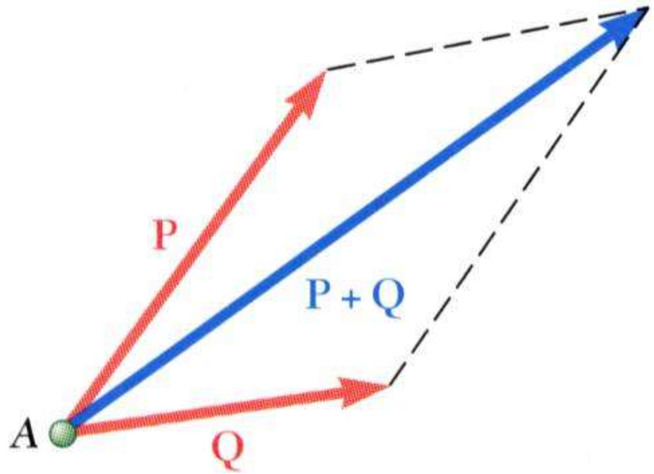
- Vector classifications:
  - *Fixed* or *bound* vectors have well defined points of application that cannot be changed without affecting an analysis.
  - *Free* vectors may be freely moved in space without changing their effect on an analysis.
  - *Sliding* vectors may be applied anywhere along their line of action without affecting an analysis.
  - *Equal* vectors have the same magnitude and direction.
  - *Negative* vector of a given vector has the same magnitude and the opposite direction


P
P + Q
A
Q

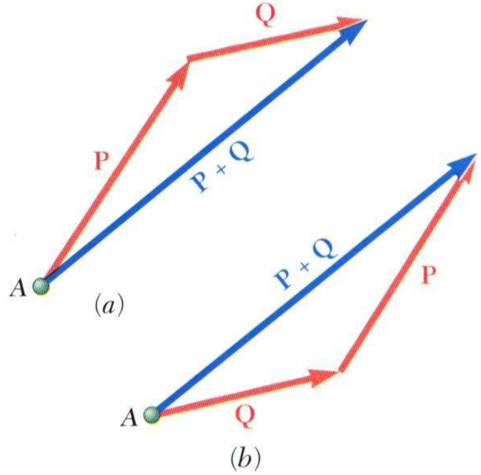

P
P


P
-P

- **Addition of Vectors**
- Trapezoid rule for vector addition
- Triangle rule for vector addition



- Law of cosines,
- Law of sines,

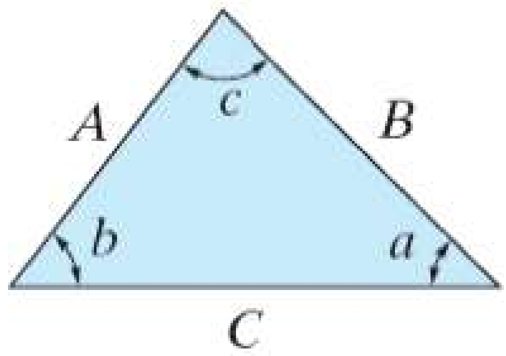


Cosine law:

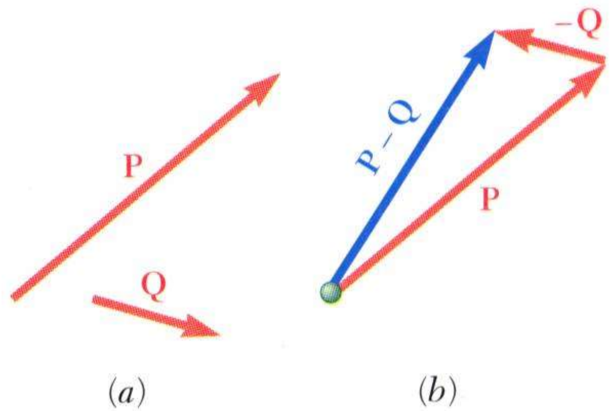
$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

Sine law:

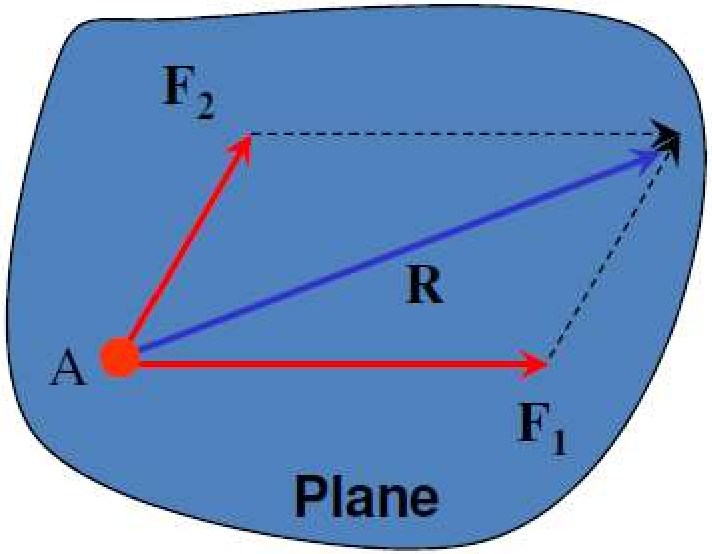
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$



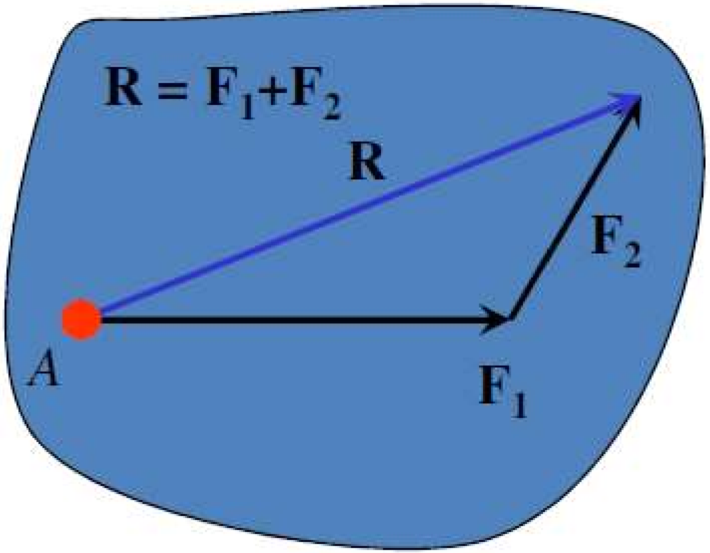
- **Addition of Vectors ……..**
- Vector addition is commutative, $\vec{P}+\vec{Q}=\vec{Q}+\vec{P}$
- Vector subtraction



- **Vector Addition….**
- Using the Parallelogram Law
- Construct a Parallelogram with two Forces as Parts. The resultant of the forces is the diagonal.

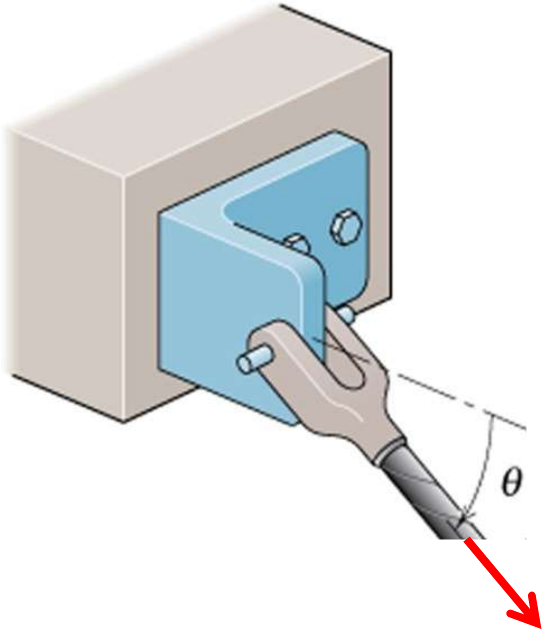


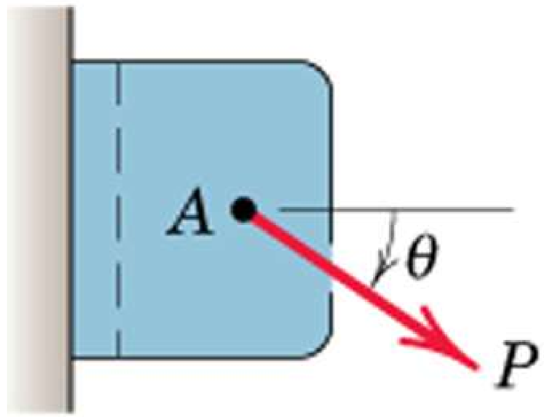
- **Vector Addition….**
- Triangle Rule:
- Draw the first Vector. Join the tail of the Second to the head of the First and then join the head of the third to the tail of the first force to get the resultant force, R



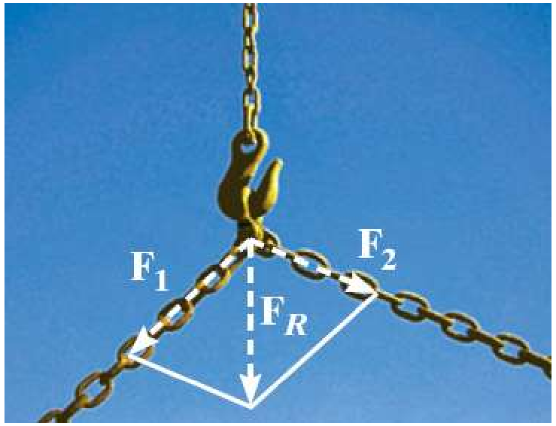
- **Force Systems**

- Force: Magnitude (P), direction (arrow) and point of application (point A) is important
- Change in any of the three specifications will alter the effect on the bracket.

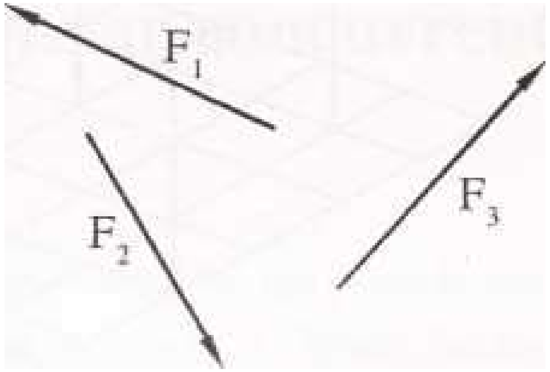




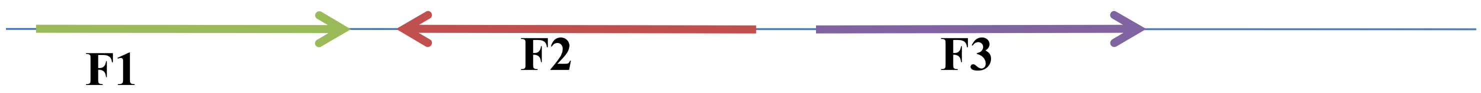
- **Force Systems....**
- **Concurrent force**



- Forces are said to be concurrent at a point if their lines of action intersect at that point
- F1, F2 are concurrent forces; R will be on same plane;
  - R = F1+F2
- Experimental evidence shows that the combined effect of two forces may be represented by a single *resultant* force.
- **Non Concurrent force**

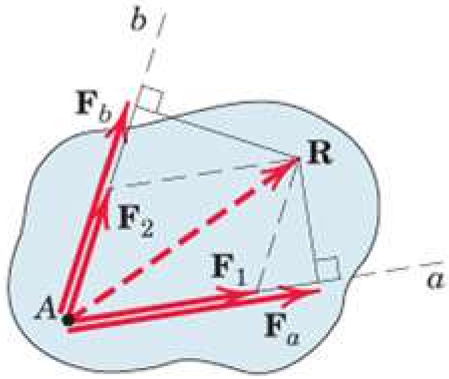


- Force Systems...
- **Collinear forces**
- Forces whose lines of action lie on the same line



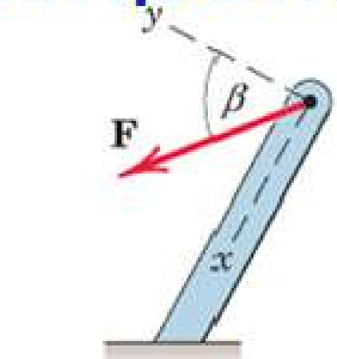
- **Like parallel and unlike parallel forces**

- **Components and Projections of Force**
- Components of a Force are not necessarily equal to the Projections of the Force unless the axes on which the forces are projected are orthogonal (perpendicular to each other).
- F1 and F2 are components of R.---- R = F1 + F2
- Fa and Fb are perpendicular projections on axes a and b, respectively.
- R ≠ Fa + Fb unless a and b are perpendicular to each other

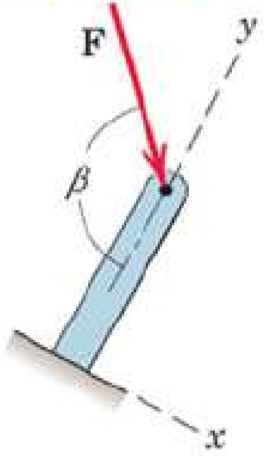


Components of Force

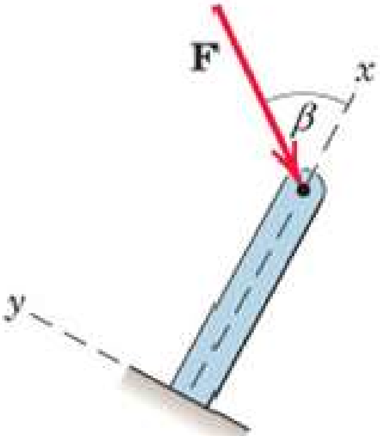
Examples



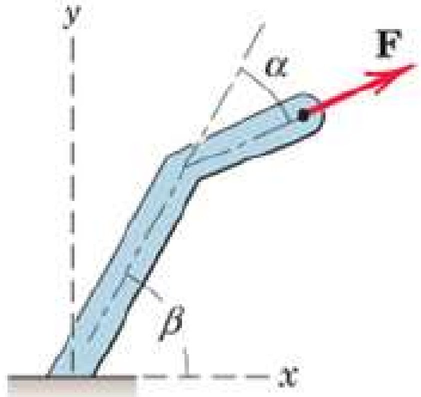
$F_x = F \sin \beta$
$F_y = F \cos \beta$



$F_x = F \sin(\pi - \beta)$
$F_y = -F \cos(\pi - \beta)$

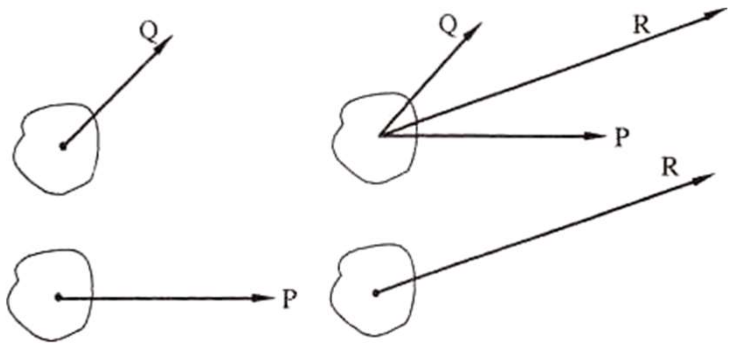


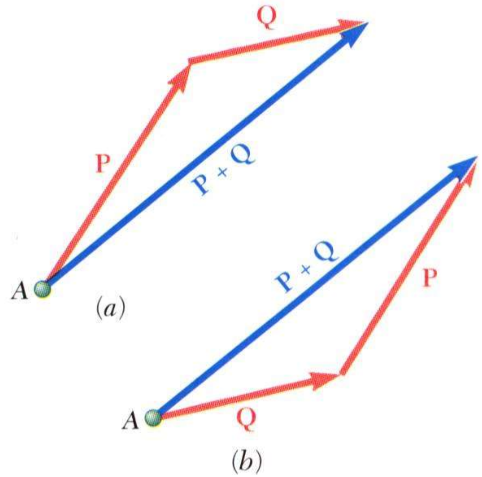
$F_x = -F \cos \beta$
$F_y = -F \sin \beta$



$F_x = F \cos(\beta - \alpha)$
$F_y = F \sin(\beta - \alpha)$

- **Coplanar Concurrent Force ...**





- Triangle Rule
- To find the vector sum of forces P and Q, draw the vector representing Q from the tip of the vector representing P. The vector connecting the tail of vector representing P and the tip of the vector representing Q, will be the vector sum of forces P and Q.
- P + Q = Q + P, ie., vector addition is commutative.

- **Parallelogram law**
- Slevinius (1548 – 1620)
- “If two forces acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces”.

- Let two forces P and Q act at a point O
- These forces P and Q are represented in magnitude and direction by the vectors OA and OB respectively
- Let the angle between the two forces be α.
- Draw the parallelogram with OA and OB as adjacent sides. The diagonal OC, according to the parallelogram law of forces, represents the magnitude and direction of the resultant R of the forces P and Q.

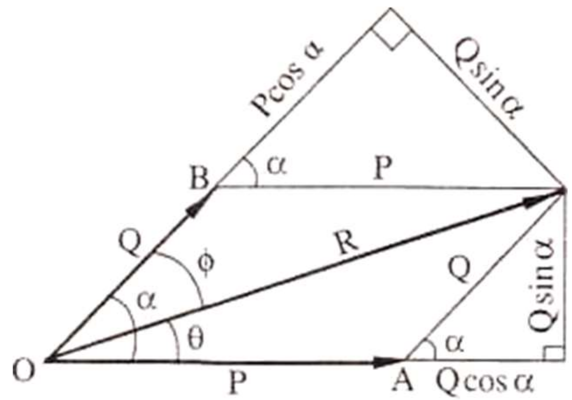


Fig. 2.4

- From C draw CD perpendicular, to OA produced. In the triangle OCD,
- $OC^2 = OD^2 + DC^2$

$= (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$

$= P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha$

$= P^2 + Q^2\cos^2\alpha + Q^2\sin^2\alpha + 2PQ\cos\alpha$

$= P^2 + Q^2[\cos^2\alpha + \sin^2\alpha] + 2PQ\cos\alpha$

$$\mathbf{R^2 = P^2 + Q^2 + 2PQ\cos\alpha}$$

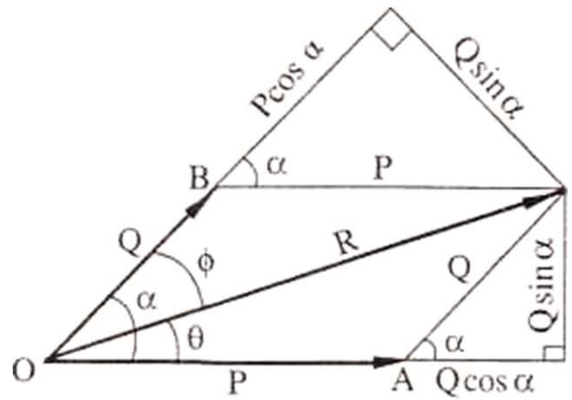


Fig. 2.4

Let θ be the inclination of resultant with force P. Then,

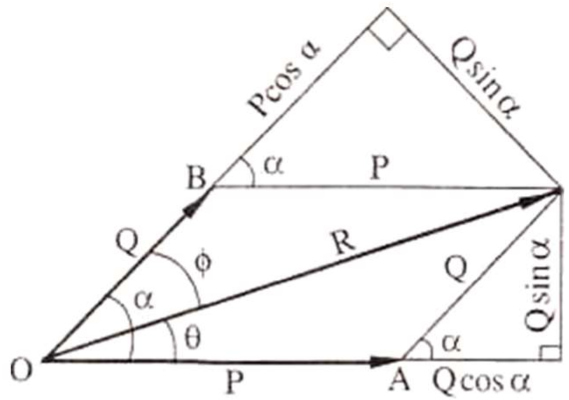


Fig. 2.4

$$\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P + Q\cos\alpha} = \frac{\sin\alpha}{\frac{P}{Q} + \cos\alpha}$$

$$\therefore \theta = \tan^{-1}\frac{\sin\alpha}{\cos\alpha + \frac{P}{Q}}$$

If ϕ is the inclination of resultant with force Q, then,

$$\phi = \alpha - \theta, \quad \text{or} \qquad \tan\phi = \frac{CE}{OE} = \frac{P\sin\alpha}{Q + P\cos\alpha} = \frac{\sin\alpha}{\cos\alpha + \frac{Q}{P}}$$

$$\therefore \phi = \tan^{-1}\frac{\sin\alpha}{\cos\alpha + \frac{Q}{P}}$$

Particular cases :

(i). when $\alpha = 0^0$

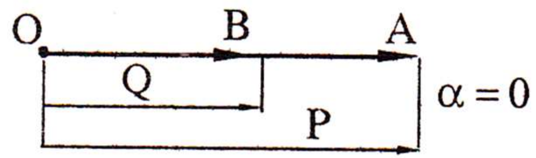
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$= \sqrt{P^2 + Q^2 + 2PQ\cos 0}$$

$$= \sqrt{P^2 + Q^2 + 2PQ}$$

$$= \sqrt{(P + Q)^2}$$

$$R = P + Q$$

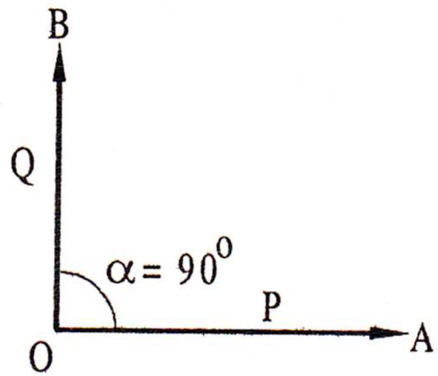


(ii) when $\alpha = 90^0$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$= \sqrt{P^2 + Q^2 + 2PQ\cos 90}$$

$$R = \sqrt{P^2 + Q^2}$$



(iii) when $\alpha = 180^0$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

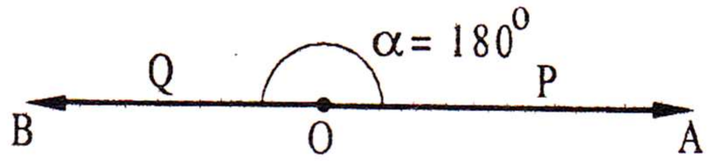
$$= \sqrt{P^2 + Q^2 + 2PQ\cos 180}$$

$$= \sqrt{P^2 + Q^2 + 2\,PQ\,(-1)}$$

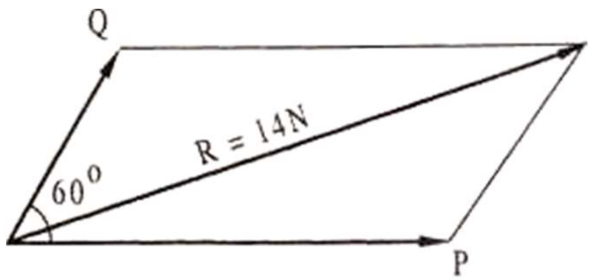
$$= \sqrt{P^2 + Q^2 - 2PQ}$$

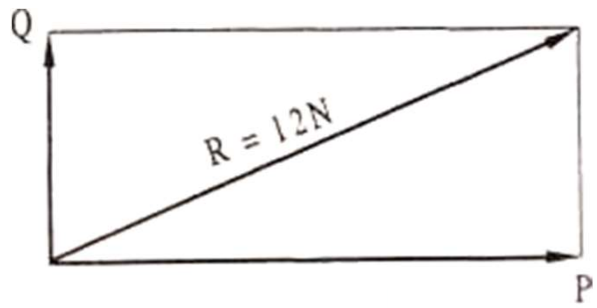
$$= \sqrt{(P - Q)^2}$$

$$R = P - Q$$



- The resultant of two forces when they act at an angle of 60° is 14N. When they act at right angles, their resultant is 12 N. Determine the magnitude of the two forces.

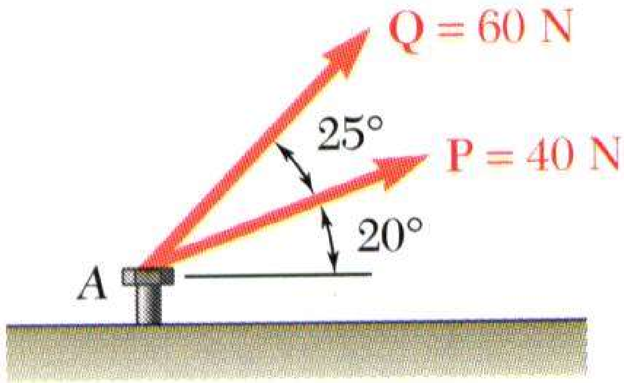




- Case(i)   R= 14 N, $\alpha = 60°$
- $R^2 = P^2 + Q^2 + 2PQ \cos \alpha$; $14^2 = P^2 + Q^2 + 2PQ. \cos 60°$
- $P^2+Q^2 + PQ= 196$ -------(i)

- Case(ii) R= 12 N, $\alpha = 90°$
- $R^2 = P^2 + Q^2$; $12^2 = P^2 + Q^2$.
- Substituting this value of $P^2 + Q^2$ in eqn (i),
- $12^2 + PQ = 196$
- **$PQ = 52$**
- $(P+Q)^2 = P^2 + Q^2 + 2PQ$
- $= 12^2 + 2 \times 52$
- $P + Q = 15.75$ N
- $(P-Q)^2 = P^2 + Q^2 - 2PQ = 144 - 2 \times 52 = 40$
- $P-Q = 6.32$ N,
- $2P = 22.07$ N
- **$P = 11.04$ N**
- **$Q = 15.75 - 11.04 = 4.71$ N**

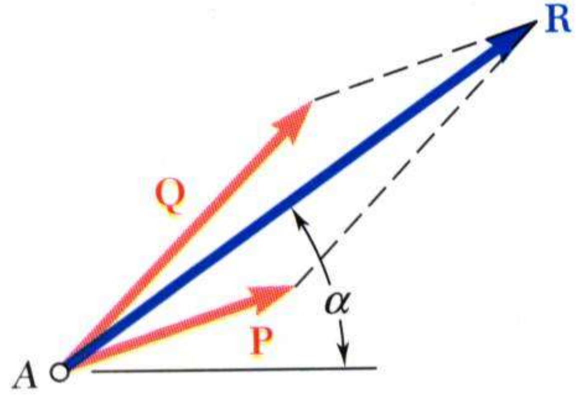
- The two forces act on a bolt at *A*. Determine their resultant.



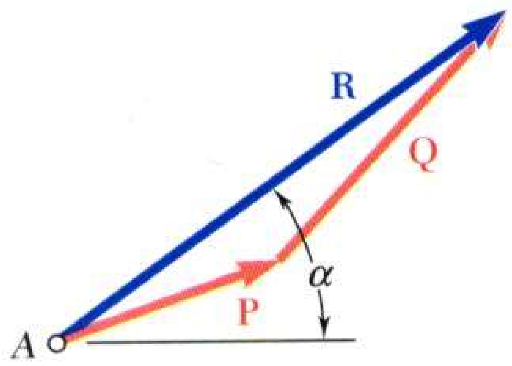
- SOLUTION:
- Graphical solution - construct a parallelogram with sides in the same direction as **P** and **Q** and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the the diagonal.
- Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant

- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured

$$\boxed{\mathbf{R} = 98\ \mathrm{N} \quad \alpha = 35^\circ}$$



- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

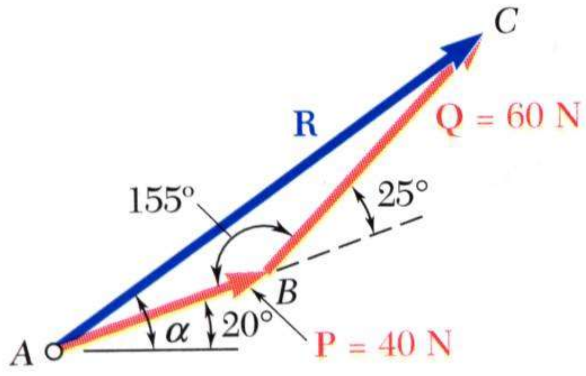


- Trigonometric solution - **Apply the triangle rule.**
  From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

$$\boxed{R = 97.73\text{N}}$$

- From the Law of Sines,



$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$
$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Cosine law:
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$
Sine law:
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

$$\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q}$$

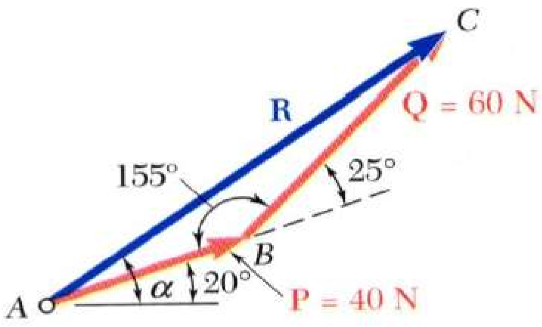
$$\boldsymbol{P} = 40[\cos(20)\boldsymbol{i} + \sin(20)\boldsymbol{j}]$$
$$= 37.58\mathbf{i} + 13.68\mathbf{j}$$

$$\boldsymbol{Q} = 60[\cos(45)\,\boldsymbol{i} + \sin(45)\boldsymbol{j}]$$
$$= 42.43\mathbf{i} + 42.43\mathbf{j}$$

$$\boldsymbol{R} = 80.01\boldsymbol{i} + 56.10\boldsymbol{j}$$

$$R = 97.72$$

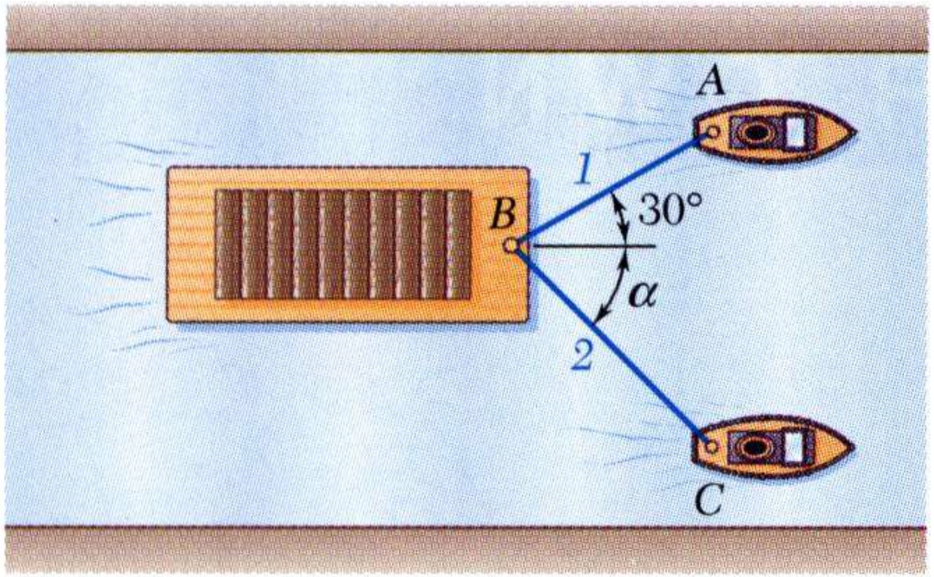
$$\alpha = 35.03^{o}$$

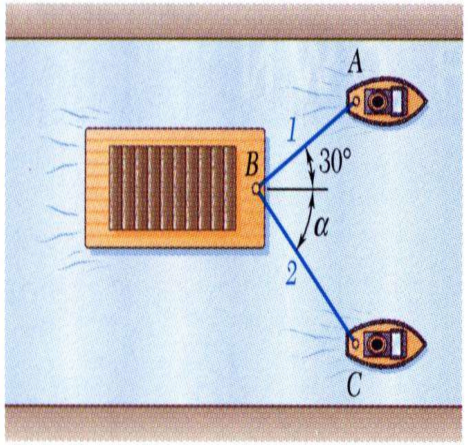


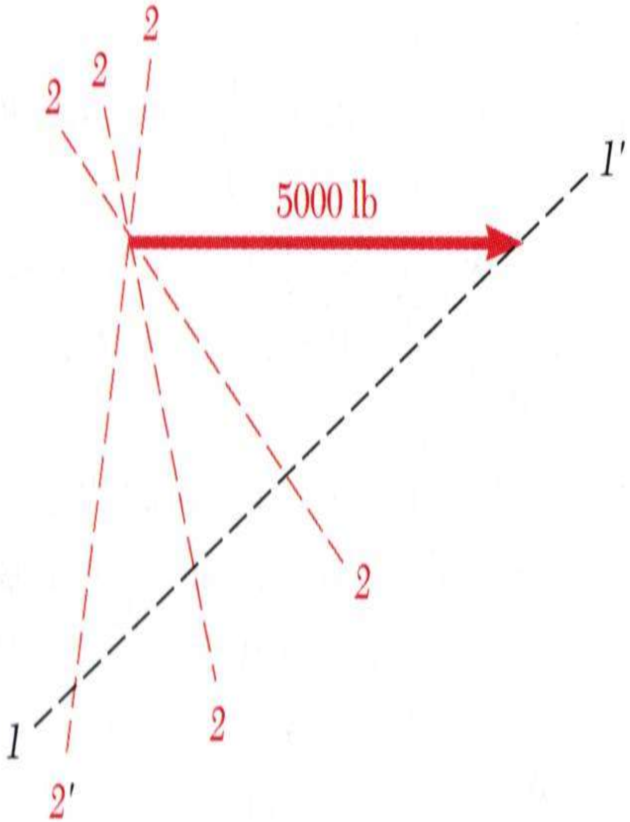
- A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 N directed along the axis of the barge, determine

a) the tension in each of the ropes for $\alpha = 45^o$,

b) the value of α for which the tension in rope 2 is a minimum.

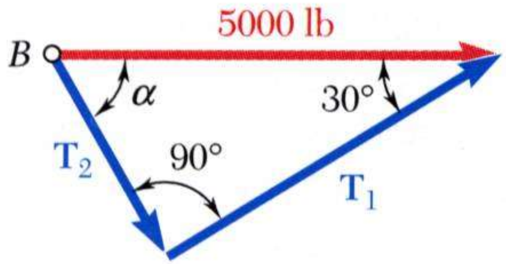






- The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in α.

- The minimum tension in rope 2 occurs when $\mathbf{T_1}$ and $\mathbf{T_2}$ are perpendicular.

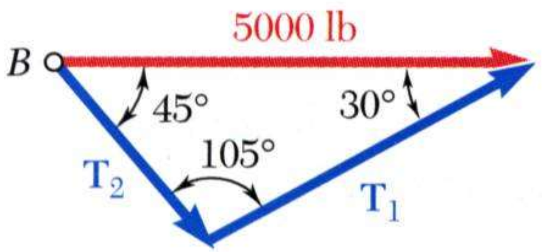


- Find a trigonometric solution by applying the Triangle Rule for vector addition. With the magnitude and direction of the resultant known and the directions of the other two sides parallel to the ropes given, apply the Law of Sines to find the rope tensions.
- The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in α.

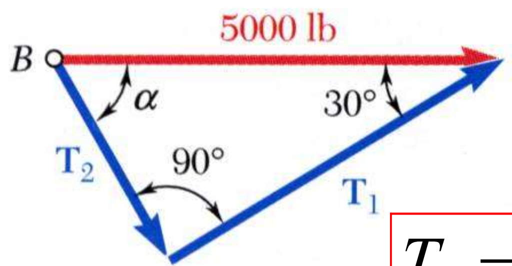
- **Trigonometric solution - Triangle Rule with Law of Sines**

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000}{\sin 105°}$$

$$\boxed{T_1 = 3660\,N \quad T_2 = 2590\,N}$$



- The minimum tension in rope 2 occurs when $\mathbf{T_1}$ and $\mathbf{T_2}$ are perpendicular.

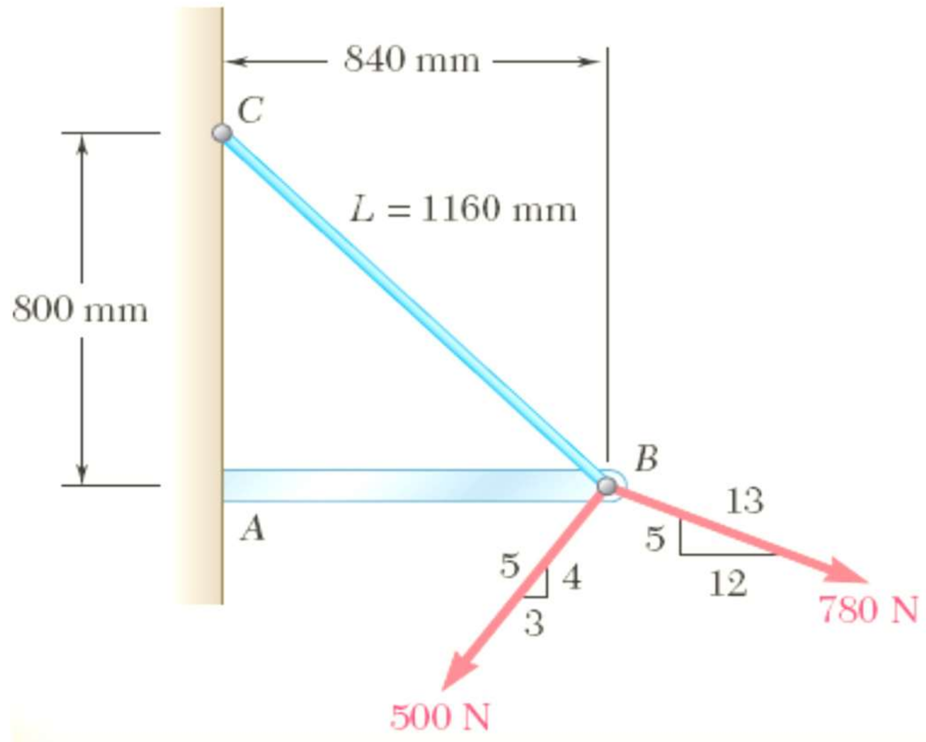


$$T_2 = (5000)\sin 30°$$

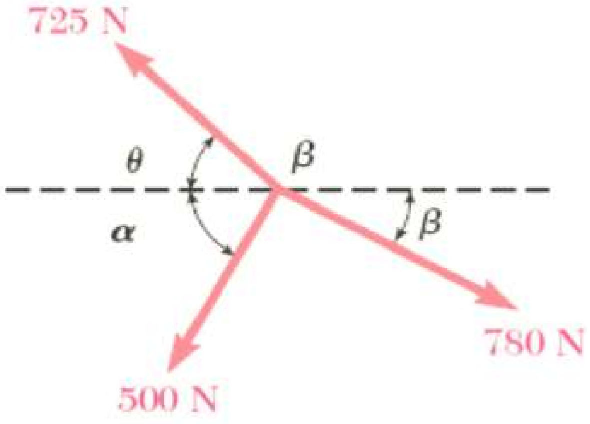
$$T_1 = (5000)\cos 30°$$

$$\boxed{T_2 = 2500\,N} \quad \boxed{T_1 = 4330\text{ N}} \quad \boxed{\alpha = 60°}$$

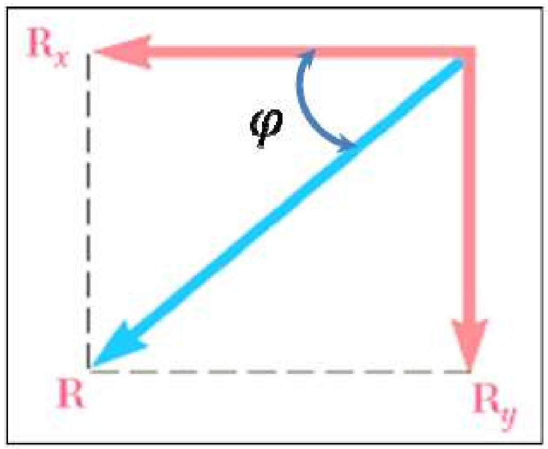
- Tension in cable BC is 725-N, determine the resultant of the three forces exerted at point B of beam AB.



- Resolve each force into rectangular components



Magnitude (N)	X-component (N)	Y-component (N)
725	-525	500
500	-300	-400
780	720	-300
	$R_x = -105$	$R_y = -200$



$$\mathbf{R} = R_x i + R_y j \qquad \mathbf{R} = (-105)i + (-200)j$$

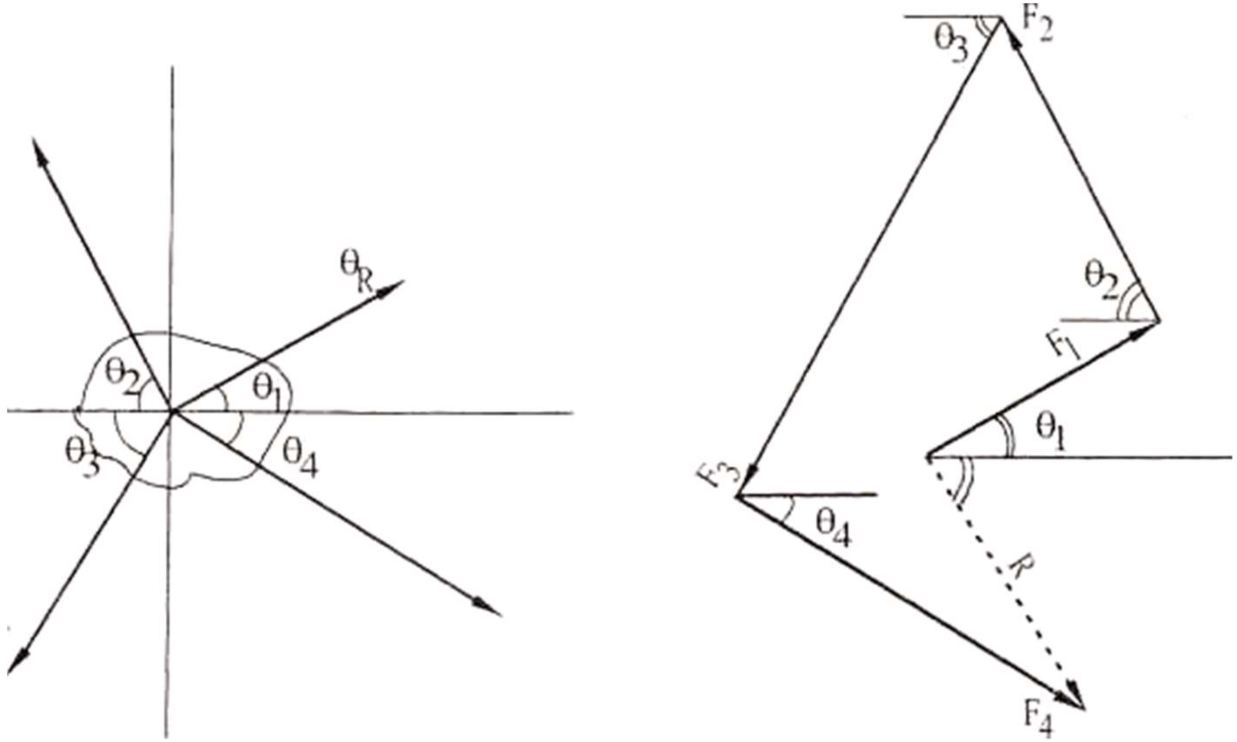
Calculate the magnitude and direction

$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62.3^o$$

$$\mathbf{R} = \sqrt{R_x^2 + R_y^2} = 225.9N$$

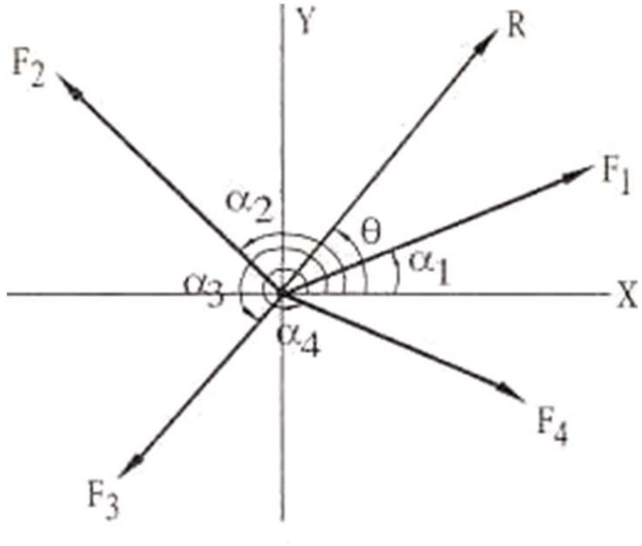
- ***Resultant of a number of coplanar concurrent forces***
- **Polygon law of forces.**
- Vector addition of a number of coplanar concurrent forces can be obtained by the repeated application of the parallelogram law.
- Repeated application of parallelogram law is the polygon law of forces
- "If a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then the resultant is represented in magnitude and direction by the closing side of the polygon, taken in the opposite order."

- **Graphical**
- Closing side of the force polygon, taken in opposite order is the resultant, R, of the concurrent forces F1,, F2, and F3 & F4,. Thus forces can be replaced by the single force R.



- **Analytical**
- Resultant of a number of coplanar concurrent forces can be obtained analytically by resolving the forces along any two mutually perpendicular directions.

- **The algebraic sum of projections of a number of coplanar concurrent forces along any direction will be equal to the projection of their resultant along the same direction.**



$$\sum F_x = R.\cos\theta \text{ ------ } (1)$$

$$F_1 \sin\alpha_1 + F_2 \sin\alpha_2 + F_3 \sin\alpha_3 + F_4 \sin\alpha_4 = R\sin\theta$$

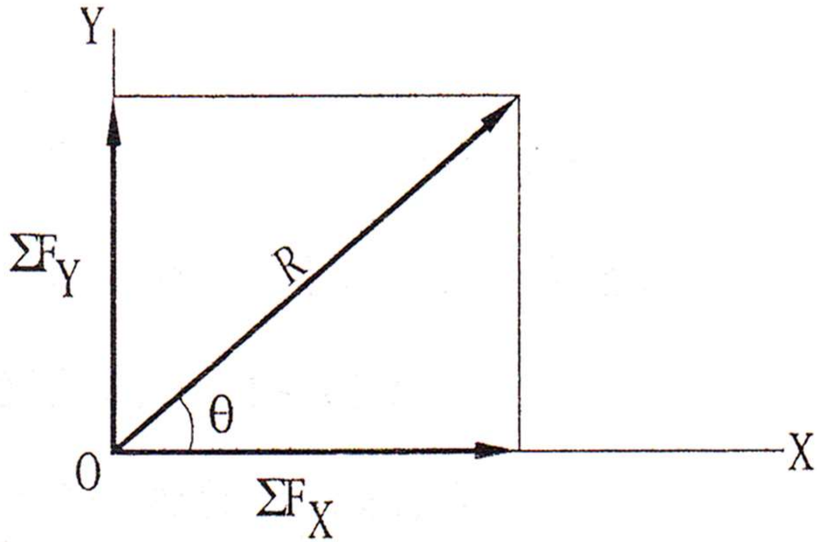
$$\sum F_y = R.\sin\theta \text{ ----- } (2)$$

Squaring and adding eqns. (1) and (2),

$$\sum F_x^{\ 2} + \sum F_y^{\ 2} = R^2\cos^2\theta + R^2\sin^2\theta$$

$$= R^2(\cos^2\theta + \sin^2\theta)$$

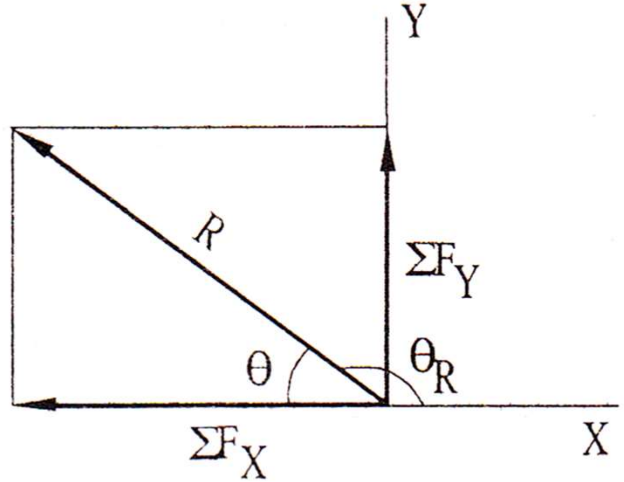
$$R = \sqrt{\sum F_x^{\ 2} + \sum F_y^{\ 2}}$$



Inclination of resultant with horizontal is given by,

$$\tan\theta = \frac{\left|\sum F_y\right|}{\left|\sum F_x\right|}$$

$$\theta = \tan^{-1}\frac{\left|\sum F_y\right|}{\left|\sum F_x\right|}$$



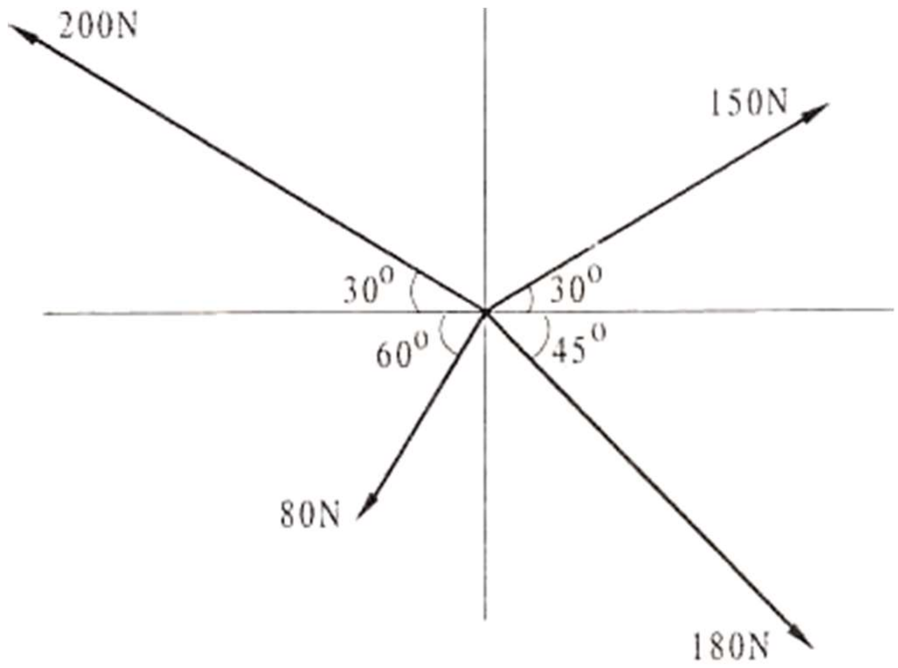
When $\sum F_x$ is - ve and $\sum F_y$ is + ve

$$\theta = \tan^{-1}\frac{\left|\sum F_y\right|}{\left|\sum F_x\right|}$$

$\theta r = 180 - \theta$

- When Σ Fx and Σ Fy are –ve
  - ❑ $\theta r = 180 + \theta$
- When Σ Fx is +ve and Σ Fy is –ve
  - ❑ $\theta r = 360 - \theta$

- Find the resultant of the forces



- Resolving the forces along
- Σ Fx = 150 cos 30 + 180 cos 45 - 200 cos 30 - 80 cos 60

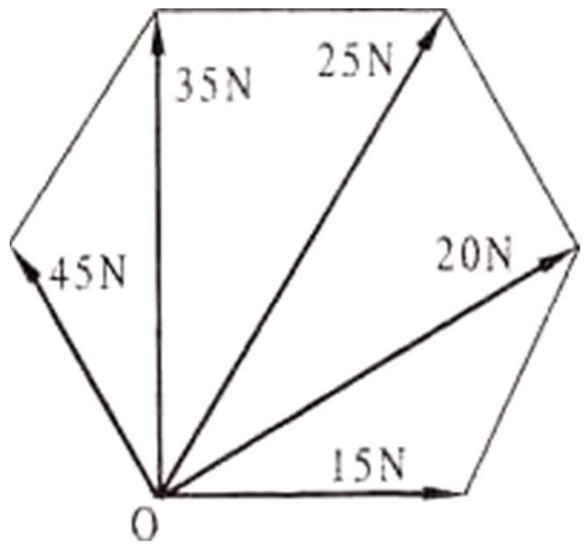
  = 43.98 N
- Σ Fy = 150 sin 30 + 200 Sin30 - 80 Sin60 - 180 sin45

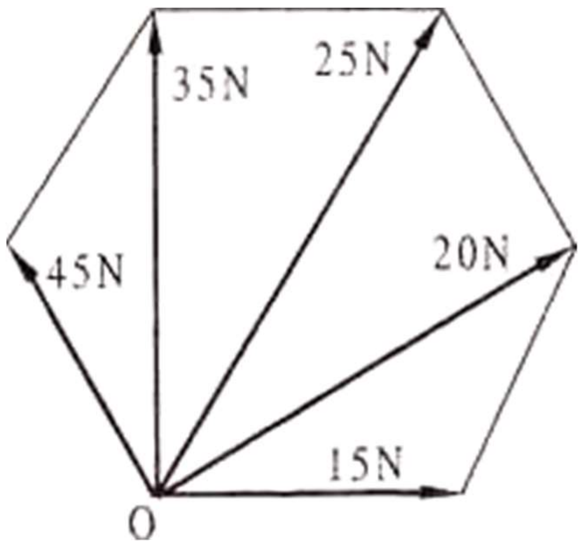
  = -21.56 N

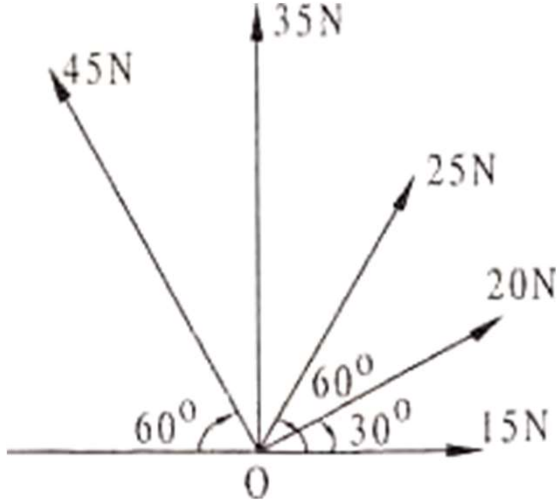
Resultant R = 48.98 N

θ r= 360 – 26.12 = 333.88

- Forces of 15 N, 20 N, 25 N, 35 N and 45 N act at an angular point of a regular hexagon towards the other angular points as shown in fig. Calculate the magnitude and direction of the resultant force.



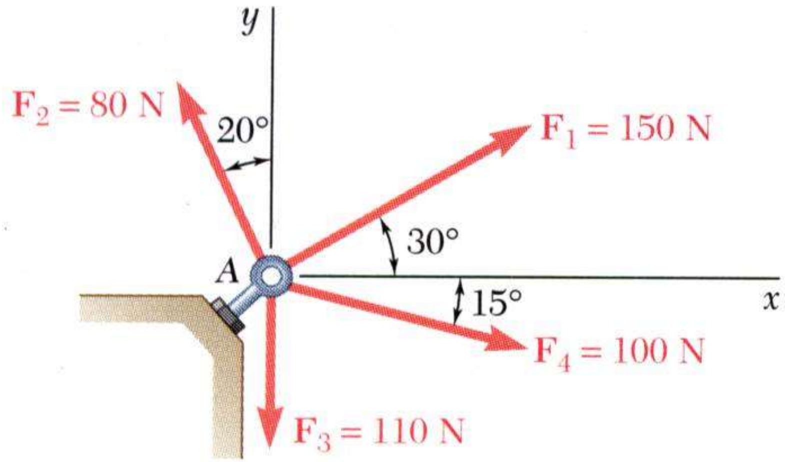




- Resultant, R = 107.95 N
- Inclination of resultant with horizontal = 78.07

- Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

force	*mag*	*x − comp*	*y − comp*
$\vec{F}_1$	150	+129.9	+75.0
$\vec{F}_2$	80	−27.4	+75.2
$\vec{F}_3$	110	0	−110.0
$\vec{F}_4$	100	+96.6	−25.9

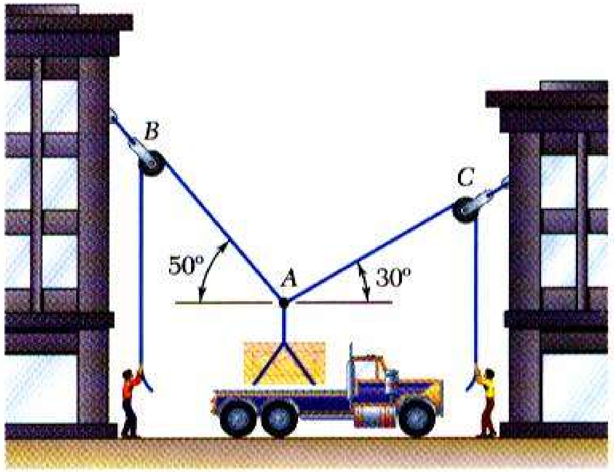


$$R_x = +199.1 \quad R_y = +14.3$$

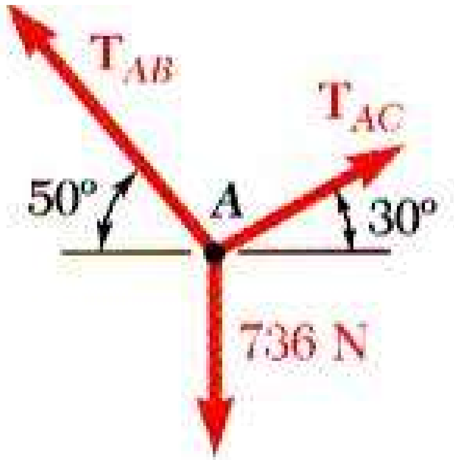
$$\boxed{R = 199.6\,\text{N}}$$

$$\tan\alpha = \frac{14.3\,\text{N}}{199.1\,\text{N}} \qquad \boxed{\alpha = 4.1°}$$

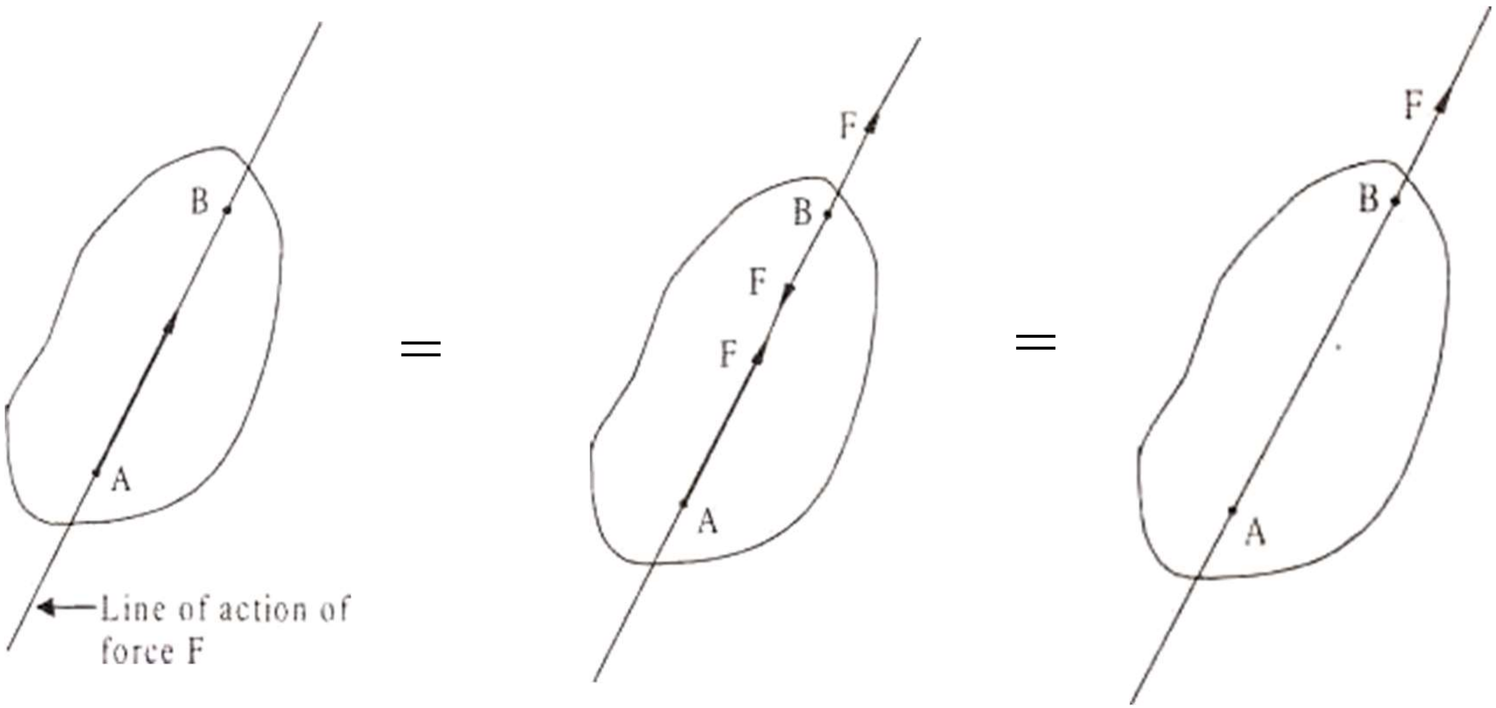
- Free-Body Diagrams
- ***Space Diagram***: A sketch showing the physical conditions of the problem.



- ***Free-Body Diagram***: A sketch showing only the forces on the selected particle.



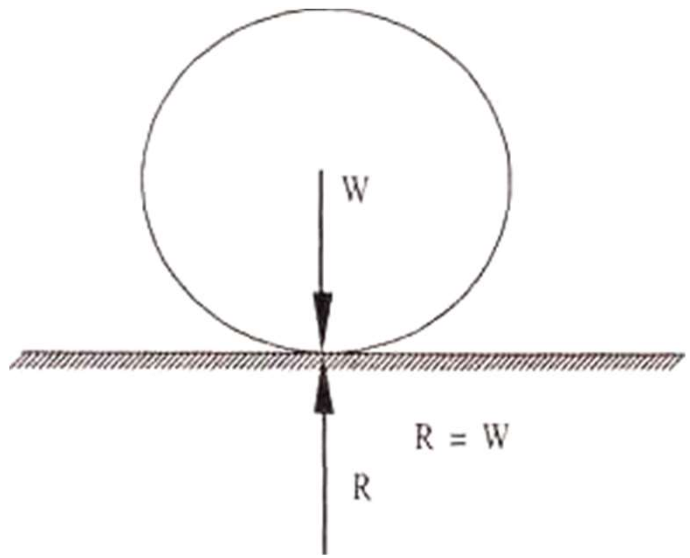
- **Principle of transmissibility**
- states that the point of application of a force can be transmitted along its line of action without changing the effect of the force on any rigid body to which it is applied



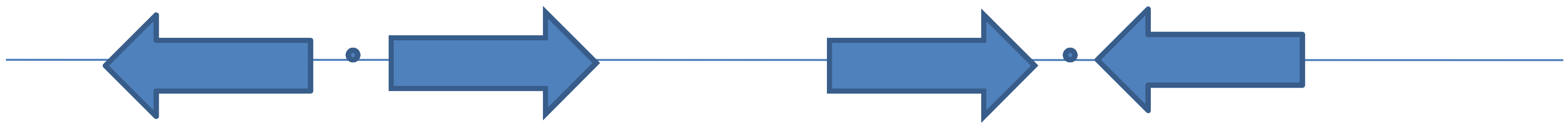
- Consider a rigid body under the action of a force F applied at A and acting along AB
- Two equal and opposite forces applied at B will not change the condition of the rigid body.
- Now the removal of force at A and the force at B which is opposite to the force at A will not change the condition of the rigid body.

- This proves that transmission of force F from its point of application at A to another point B which is in the line of action of force F does not change the condition of the rigid body.

- Law of action and reaction



- Compression & Tension members.



- **Equations of Equilibrium**

- ***A number of forces acting on a particle is said to be in equilibrium when their resultant force is zero.***

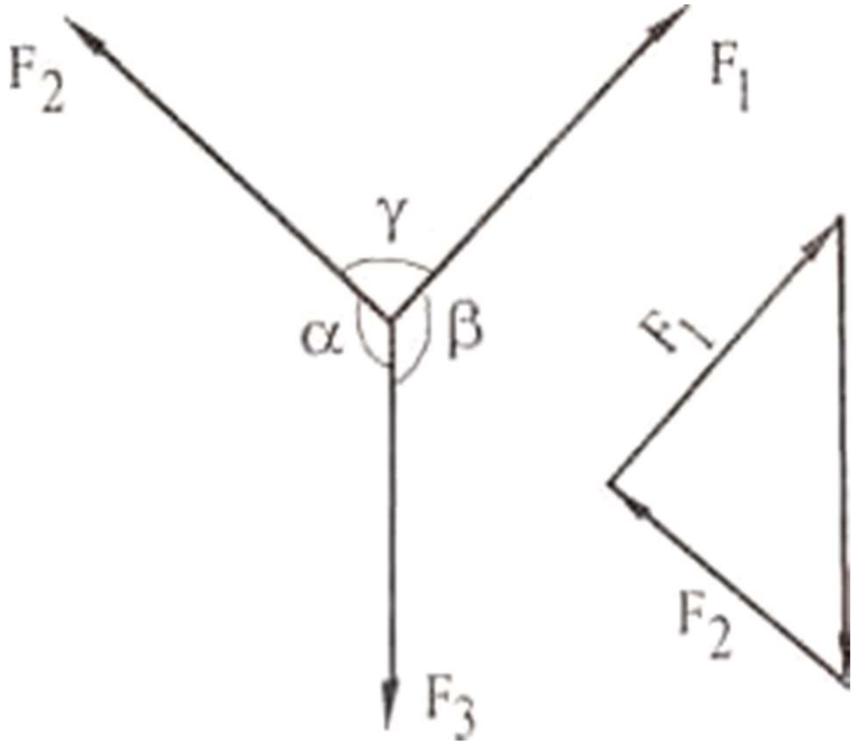
- If the resultant force is not equal to zero, then the particle can be brought to rest by applying a force equal and opposite to the resultant force. Such a force is called equilibriant.
- Resultant and equilibriant are equal in magnitude and opposite in direction.

- **Equations of Equilibrium ….**

- Resultant of a number of coplanar concurrent forces is given by
- $R = [(\Sigma\ Fx)^2 + (\Sigma\ Fy)^2]^{1/2} = 0$
- For R to be zero, Σ Fx & Σ Fy has to be zero
- So **Equations of Equilibrium are**
- **Σ Fx & Σ Fy = 0**

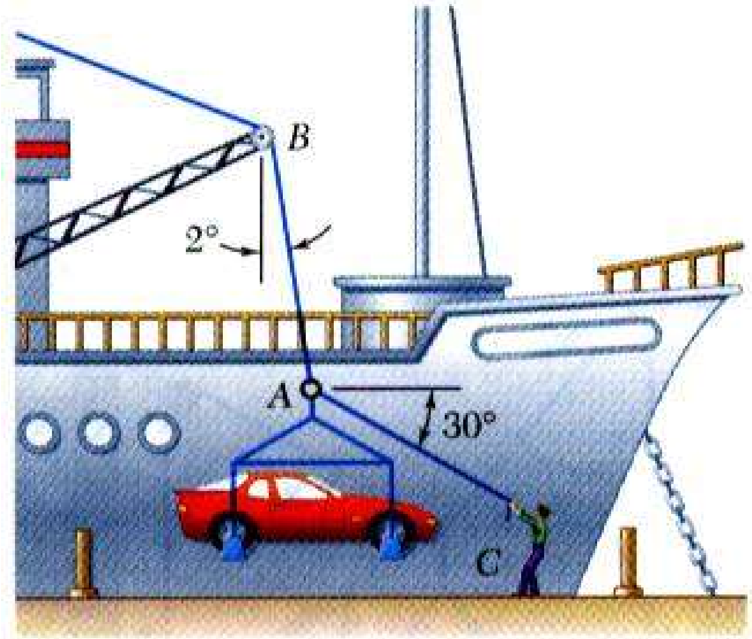
- **Lami's theorem**
- Consider three coplanar, concurrent forces F_1 , F_2 and F_3
- three forces will be in equilibrium if their resultant is zero.
- *For this the force polygon must be a closed one. ie., the force polygon must be a triangle.*
- The magnitude and direction of forces should be such that each force must be proportional to the sine of angle between the other two forces.
- **If three coplanar concurrent forces are in equilibrium, then each force is proportional to the sine of the angle between the other two forces**

- **Lami's theorem…**



$$\frac{F_1}{\sin\ \alpha} = \frac{F_2}{\sin\ \beta} = \frac{F_3}{\sin\ \gamma}$$

- In a ship-unloading operation, a 3500 N automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?



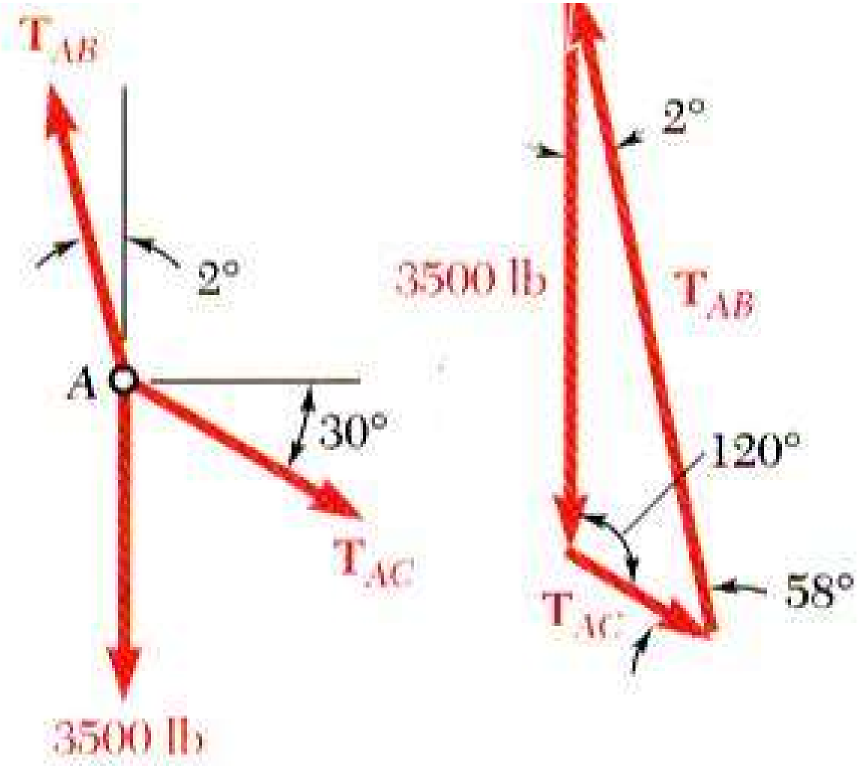
- Construct a free-body diagram for the particle at the junction of the rope and cable.
- Apply the conditions for equilibrium by creating a closed polygon from the forces applied to the particle.
- Apply trigonometric relations to determine the unknown force magnitudes.

- Solve for the unknown force magnitudes.

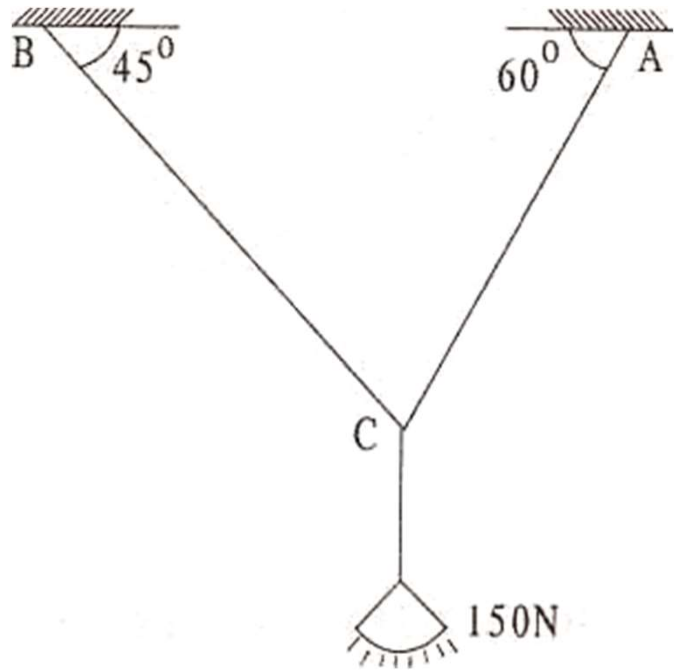
$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{3500}{\sin 58°}$$

$$T_{AB} = 3570\text{N}$$

$$\boxed{T_{AC} = 144\text{N}}$$



- An electric light fixture weighing 150 N hangs from a point C by two stay wires AC and BC as shown. Determine the tensions in the stay wires AC and BC using Lami's theorem. Verify the answer using any other method.

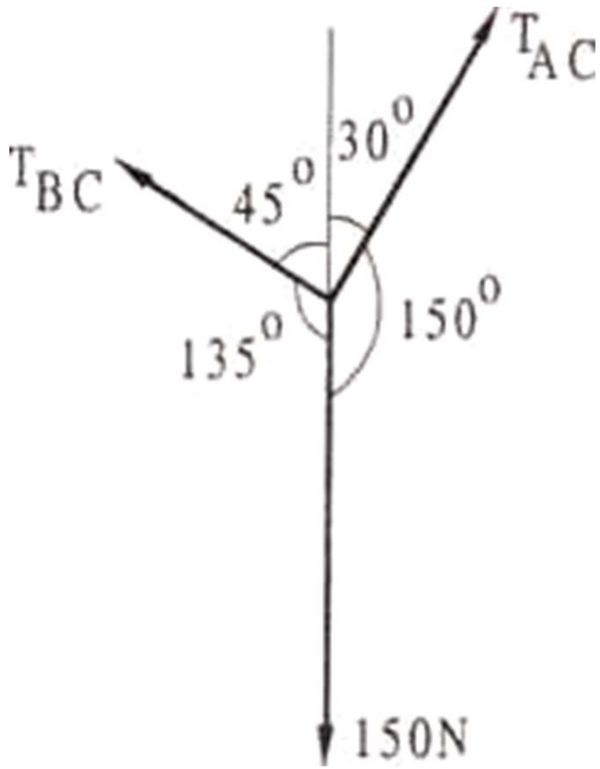


- Using Lami's theorem

$$\frac{150}{\sin 75°} = \frac{T_{BC}}{\sin 150°} = \frac{T_{AC}}{\sin 135°}$$

$$T_{BC} = 77.65N$$

$$T_{AC} = 109.81N$$

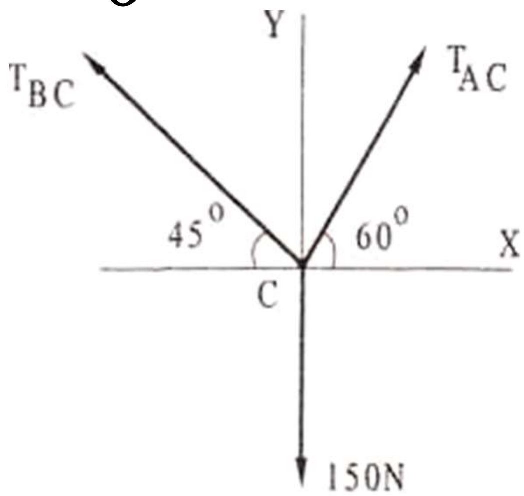


- Using method of projections.
- **Σ Fx = 0**

$$T_{AC}\cos 60° - T_{BC}\cos 45° = 0$$

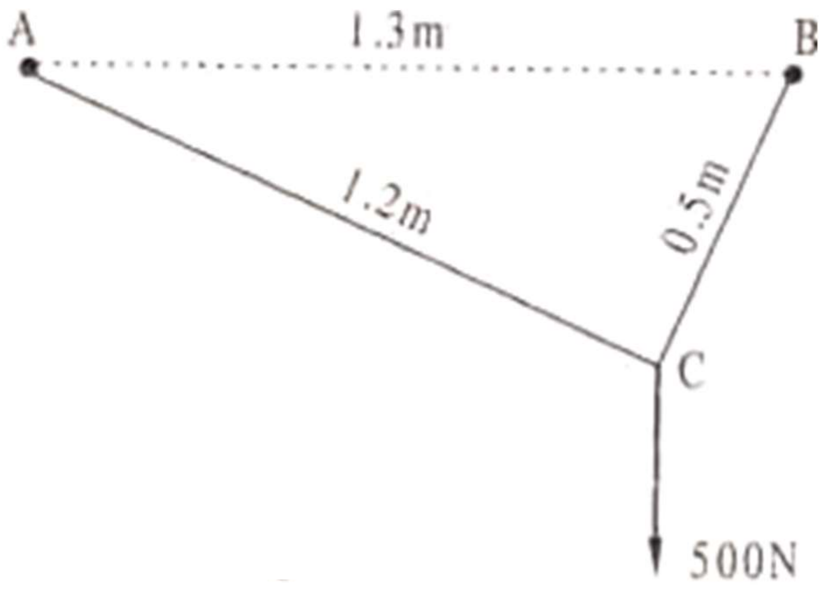
- **Σ Fy = 0**

$$T_{AC}\sin 60° + T_{BC}\sin 45° - 150 = 0$$



- **Solve two equations and get answ.**

- Two cables AC and BC are tied together at the point C to support a load of 500 N at C. A and B are at a distance of 1.3 m and are on the same horizontal plane. AC and BC are 1.2 m and 0.5 m respectively. Find the tensions in AC and BC.



- $AC^2 + BC^2 = 1.2^2 + 0.5^2 = 1.69 = 1.3^2 = AB^2$
- Since $AC^2 + BC^2 = AB^2$, angle ACB = 90°
- $\theta + \Phi = 90°$
- $1.3 \cos \theta = 1.2$; $\theta = 22.62°$;
- $\Phi = 90 – 22.62 = 67.38°$
- Applying Lami's theorem,

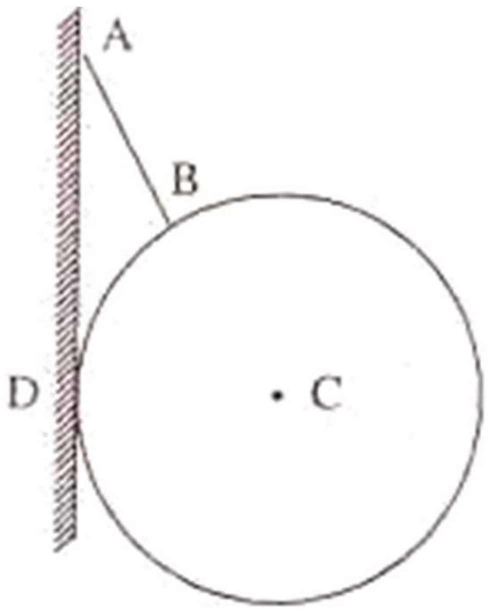
$$\frac{500}{\sin(\theta+\varphi)} = \frac{T_{AC}}{\sin(180-\theta)} = \frac{T_{AC}}{\sin(180-\varphi)}$$

$$T_{BC} = 192.31N$$

$$T_{AC} = 461.54N$$

- A smooth sphere is supported in contact with a smooth vertical wall by a string fastened to a point in its surface, the other end being attached to a point in the wall. If the length of the string be equal to the radius of the sphere find the inclination of the string to the vertical, the tension in the string and the reaction of the wall.

- Let T be the tension in the string and R be the reaction of the wall. Let the weight of sphere be W.
- $\cos\theta = r/(2r) = 1/2$



$$\therefore \theta = 60^0$$

For $\sum F_y = 0$,

$$T \sin\theta - W = 0$$

$$T = \frac{W}{\sin 60} = 1.154\ W$$

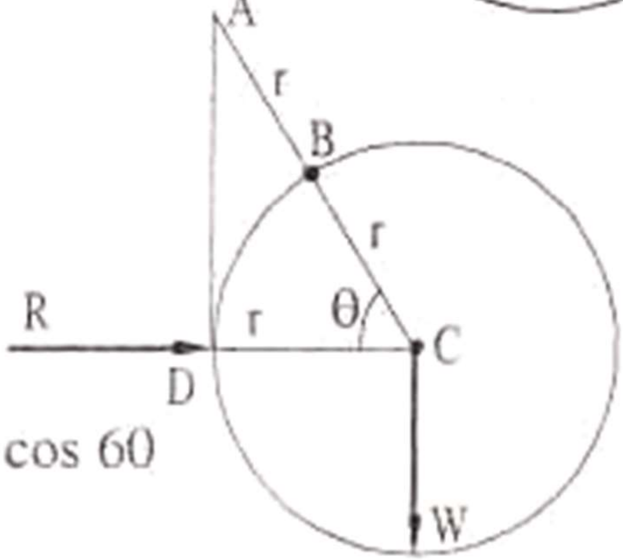
For $\sum F_x = 0$

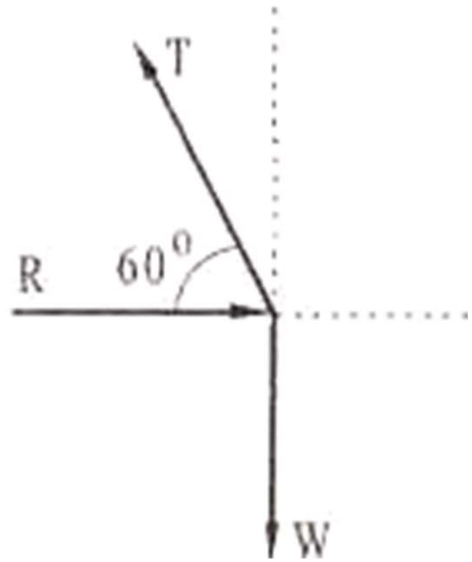
$$T \cos 60 - R = 0$$

$$R = T \cos 60$$

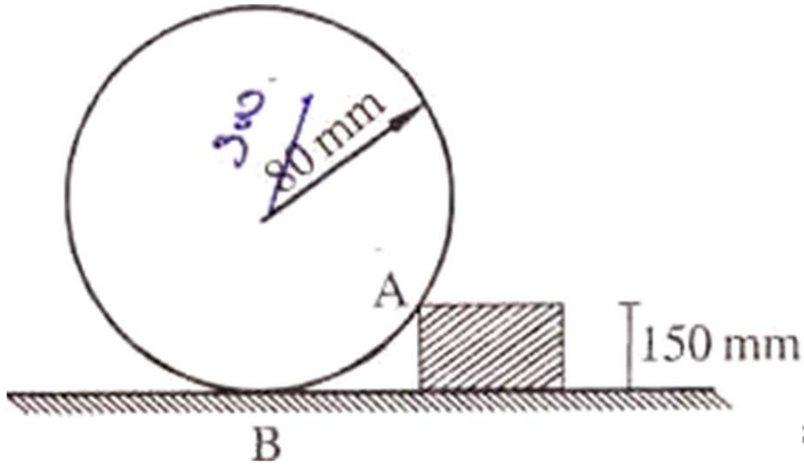
$$= 1.154\ W \times \cos 60$$

$$R = 0.577\ W$$

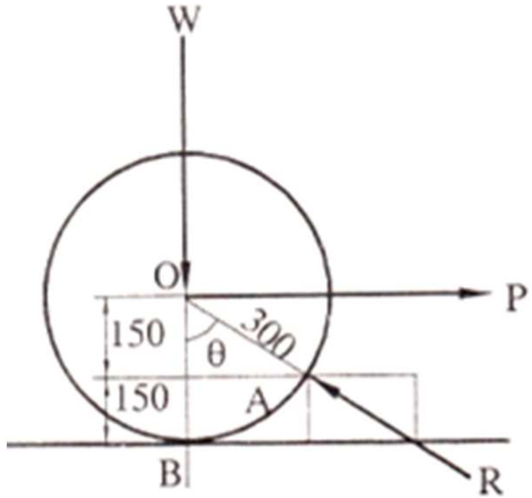


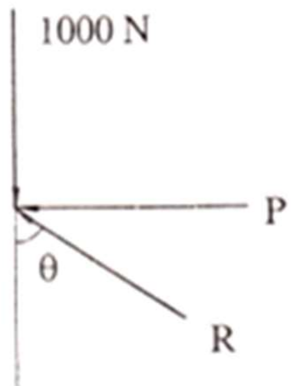


- A roller of radius 300 mm and weight 1000 N is to be pulled over a rectangular block of height 150 mm as shown in fig. Determine
- (i) the horizontal force required to be applied through the centre O.
- (ii) the least force required to be applied through the centre O and
- (iii) the required horizontal force when it is applied through the top end of vertical diameter.



- Case (i), a horizontal force is applied through the centre.
- When the roller is just turned about A, the contact at B brakes and hence there is no reaction at B. Let P be the applied force and R be the reaction at the contact point A.
- OB = OA cos θ +150
- 300 = 300 cos θ + 150
- θ = 60°;
- Resolving the forces vertically,
- R cos θ - 1000 = 0
- R = 2000 N
- Resolving the forces horizontally,
- P – R Sinθ = 0
- P = 2000 sin 60 = 1732.05 N

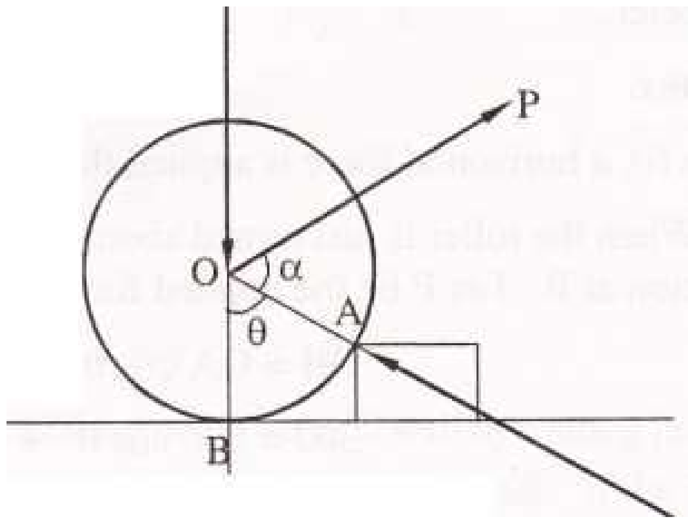




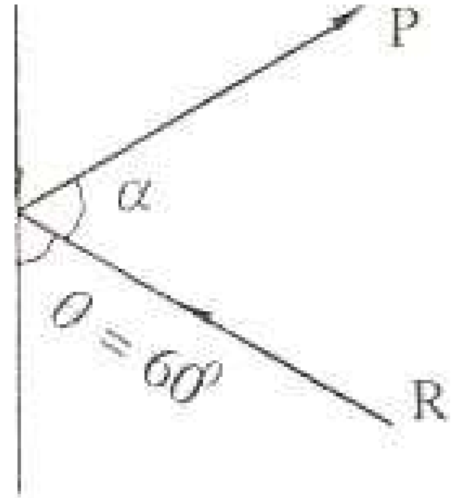
- Case (ii), the least force required when it is applied through the centre.
- Let α be the inclination of applied force with OA
- Applying Lami's theorem

$$\frac{1000}{\sin\alpha} = \frac{P}{\sin 120} = \frac{R}{\sin(180-(\alpha+60))}$$

$$P = \frac{\sin 120 x 1000}{\sin\alpha}$$

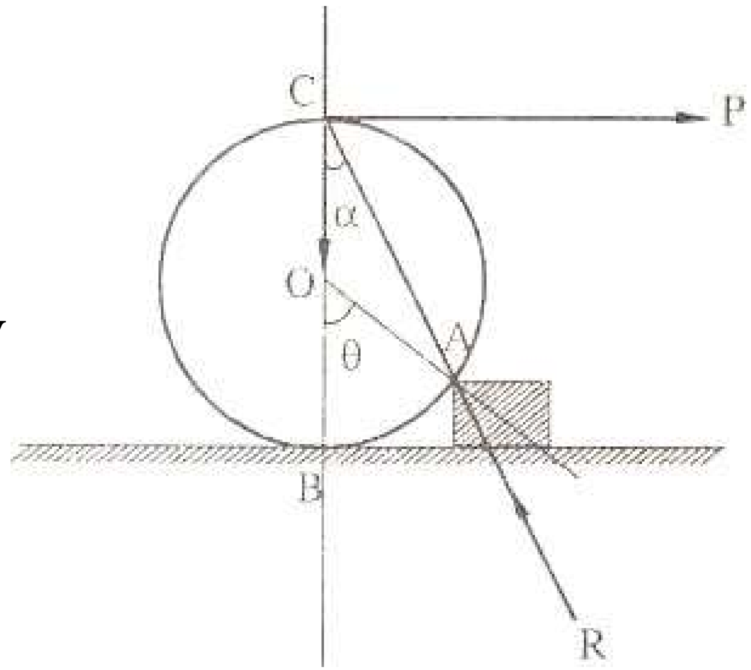


- For P to be minimum, the denominator of the above expression ie., sin α must be maximum , α = 90

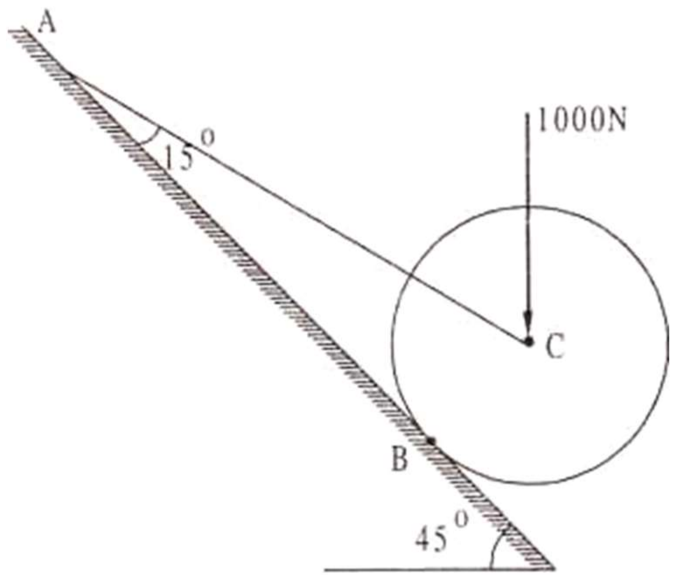


$$P = \frac{\sin 120 x 1000}{\sin 90} = 866$$

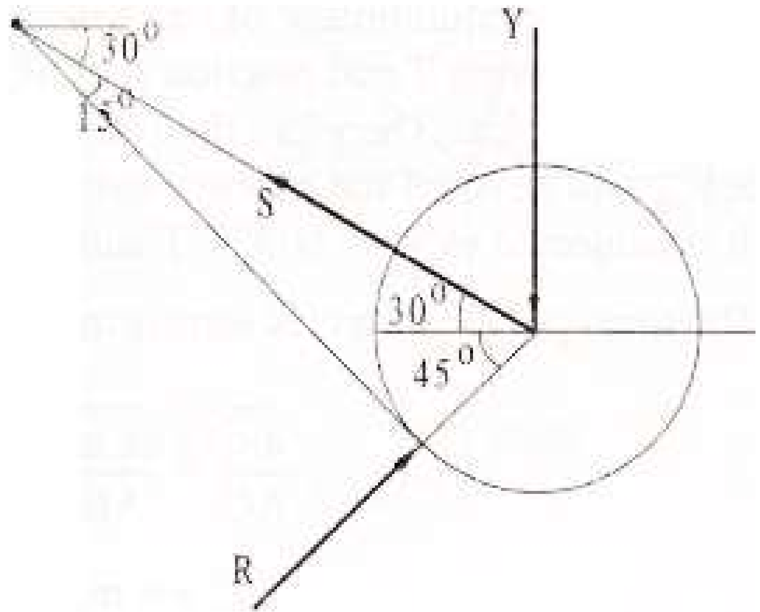
- Case (iii), when the force P is applied through the top end of the diameter.
- The line of action of R should intersect at C, where the line of action of other two forces intersect. Triangle OAC is **an isosceles triangle** with < AOC = 120°
- α= (180 -120) /2 = 30
- Resolving the forces vertically
- R cos 30 - W = 0 ; R = 1154.7
- Resolving the forces horizontally
- P- R sin 30 = 0 ;
- P = 577.35 N



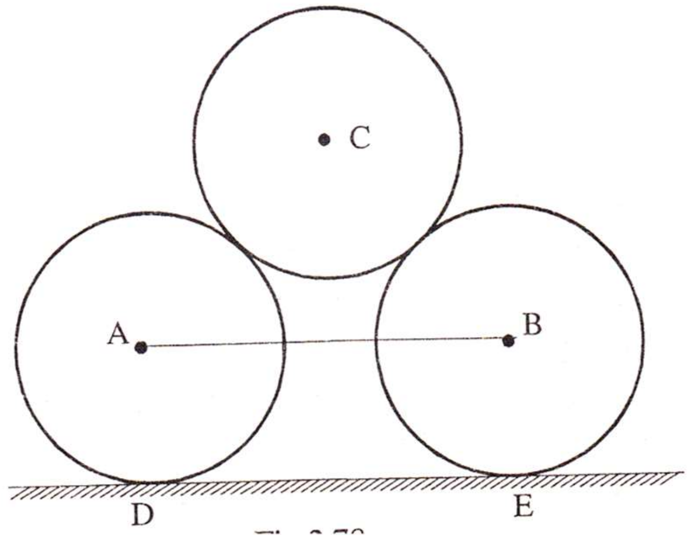
- A roller of weight W = 1000 N rests on a smooth inclined plane and is kept from rolling down by a string AC as shown in fig. Using method of projections, find the tension S in the string and the reaction at the point of contact B.



- Let S be the tension in the string and R be the reaction at B.
- Resolving the forces along X direction, $\Sigma Fx = 0$
- $R \cos 45 - S \cos 30 = 0$
- Resolving the forces along Y direction, $\Sigma Fy = 0$
- $R \sin 45 + S \sin 30 = 1000$
- $R = 896.57$ N
- $S = 732.05$ N

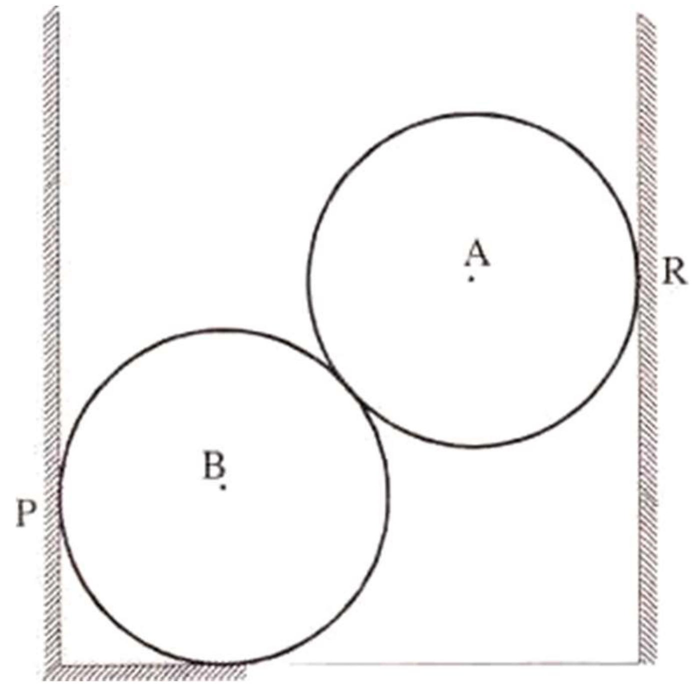


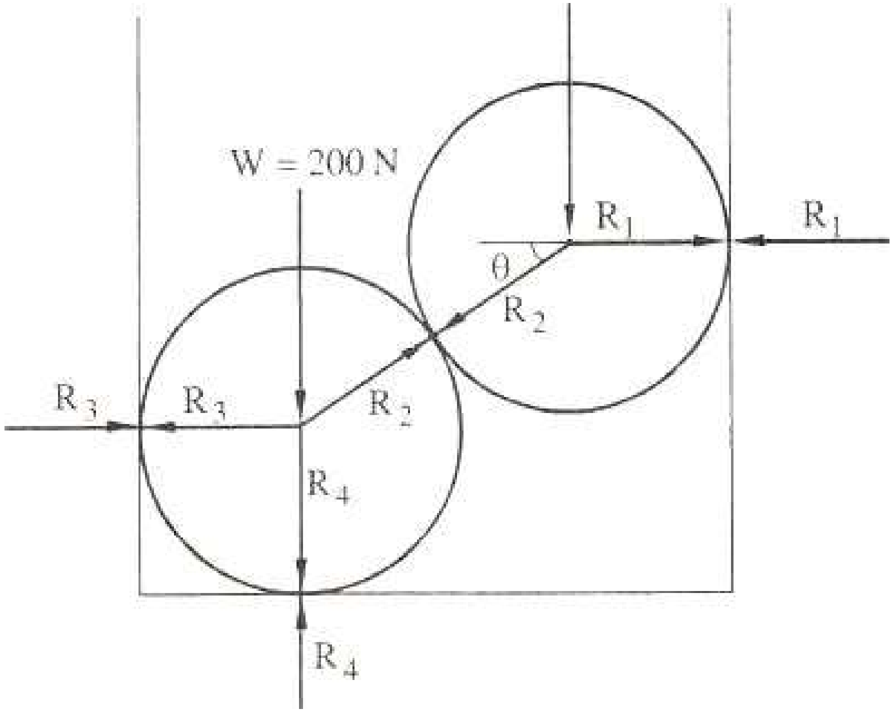
- Two smooth circular cylinders each of weight 100 N and radius 15 cm are connected at their centres by a string AB of length 40 cm and rest upon a horizontal plane as shown in fig. The cylinder above them has a weight 200 N and radius of 15 cm. Find the force in the string AB and the pressure produced in the floor at the points of contact D and E.

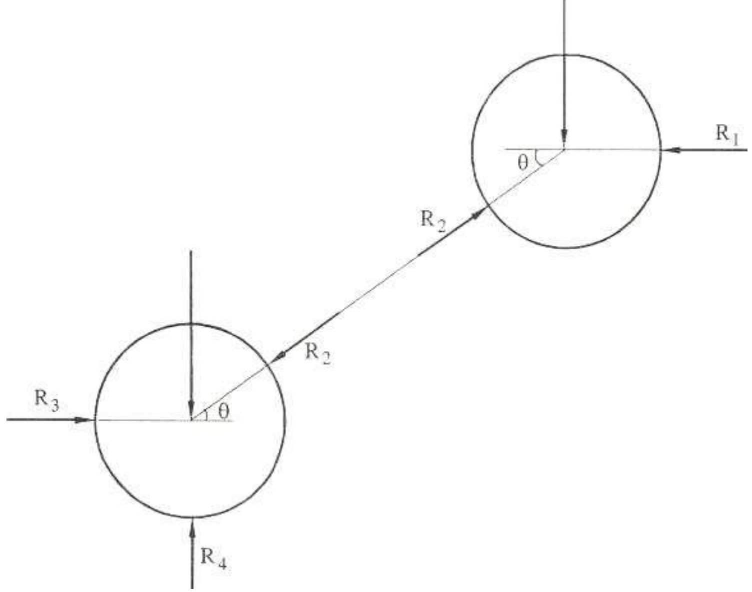


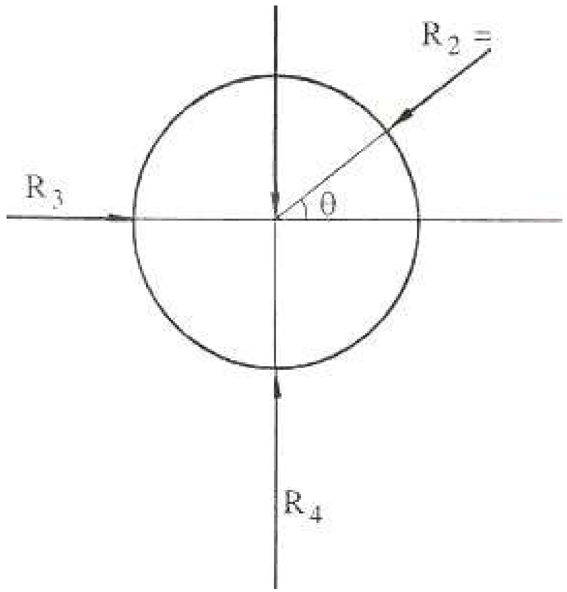
- Draw free body diagram for each sphere
- Let Rd and Re be the reactions and S be the tension. R1 and R2 be the reaction exerted by C on A & B
- R1 = R2 = 134.16 N
- S = 89.44 N
- Rd = 200 N
- Re= 200 N

- Two smooth cylinders A and B each of diameter 400 mm and weight 200 N rest in a horizontal channel having vertical walls and base width of 720 mm as shown in fig Find the reaction at P, Q and R.
- AB = 400 mm
- BC = 720 - 400 = 320 mm
- $\theta = 36.87$




W = 200 N
R1
R1
θ
R2
R3
R3
R2
R4
R4


R1
θ
R2
R2
R3
θ
R4


R2 =
R3
θ
R4