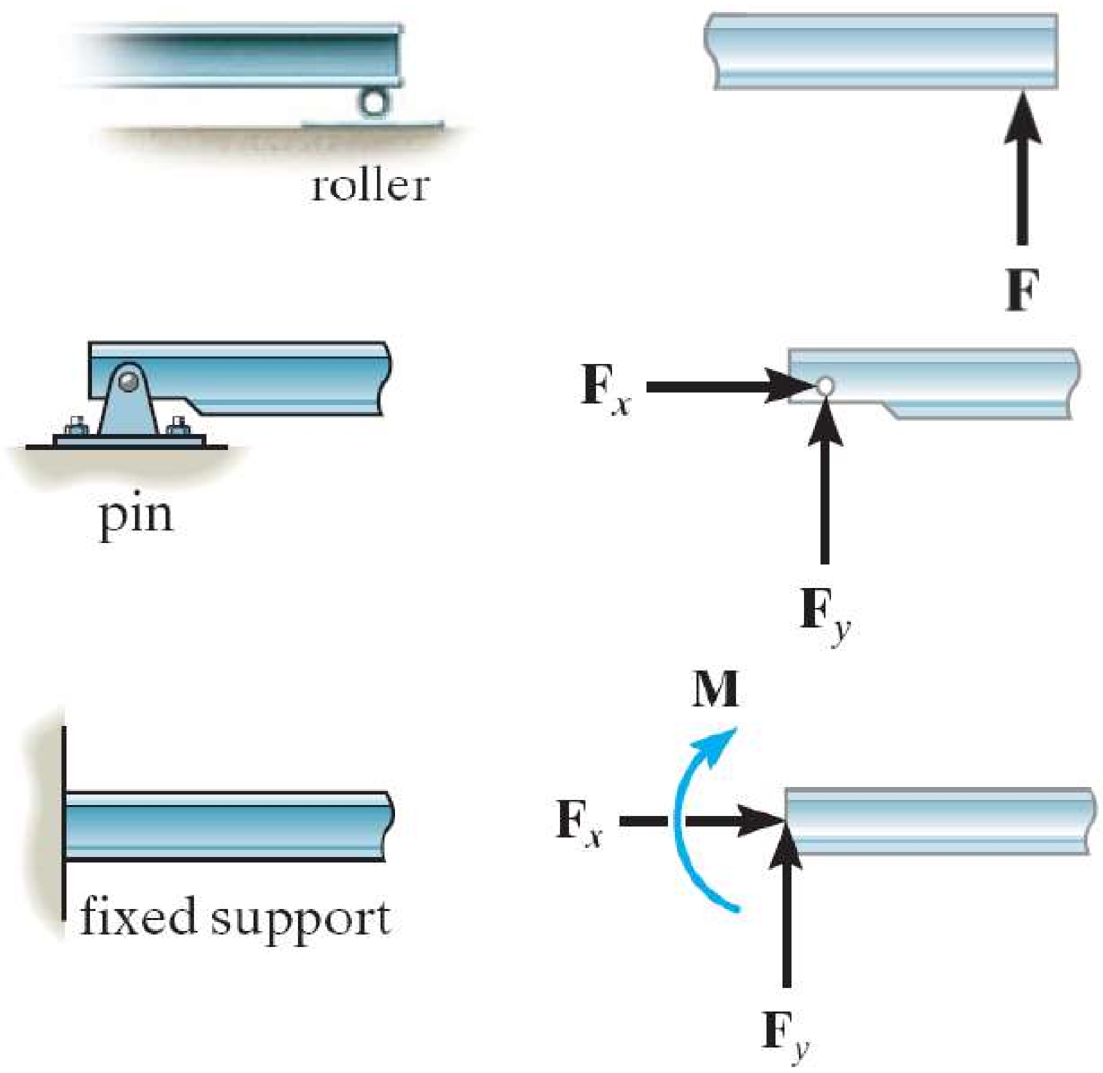
Module 2(contd…)

- **Module 2**
- Types of supports –
  - Problems involving point loads and uniformly distributed loads only.

- Force systems in space
  - Degrees of freedom –
  - Free body diagram –
  - Equations of equilibrium –
  - Simple resultant and Equilibrium problems.

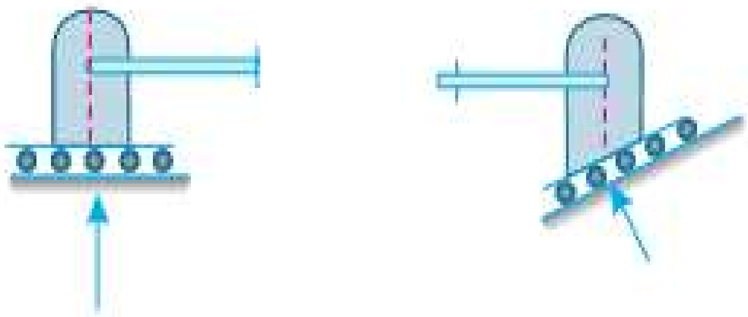
- **Equilibrium**
- **Equilibrium of a body is a condition in which the resultants of all forces acting on the body is zero.**

- Types of Supports



12.14. ROLLER SUPPORTED BEAMS

In such a case, the end of a beam is supported on rollers, and the reaction on such an end is always *normal to the support*, as shown in Fig. 12.17 (*a*) and (*b*). All the steel trusses, of the bridges, have one of their ends as supported on rollers.



(*a*) (*b*)

Fig. 12.17. Roller supported end

The main advantage, of such a support, is that the beam can move easily towards left or right, on account of expansion or contraction due to change in temperature.

12.15. HINGED BEAMS

In such a case, the end of a beam is hinged to the support as shown in Fig. 12.18. The reaction on such an end may be *horizontal*, *vertical* or *inclined*, depending upon the type of loading. All the steel trusses of the bridges have one of their end roller supported, and the other hinged.

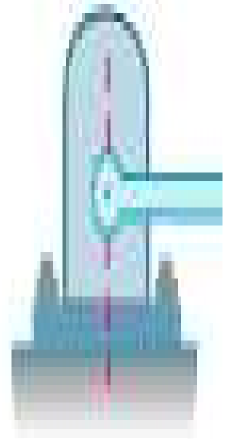
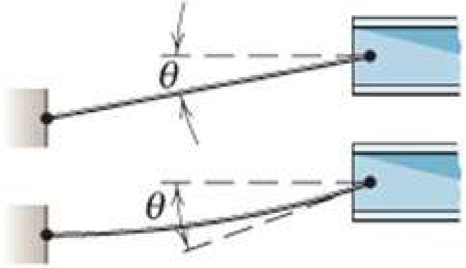
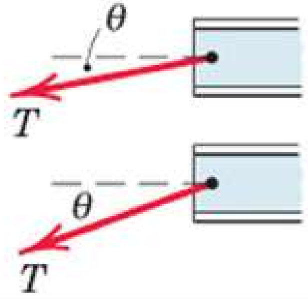
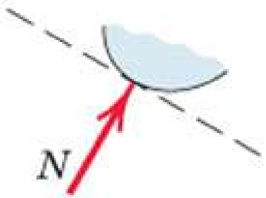
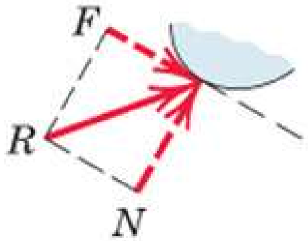


Fig. 12.18. Hinge

- Various Supports

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant

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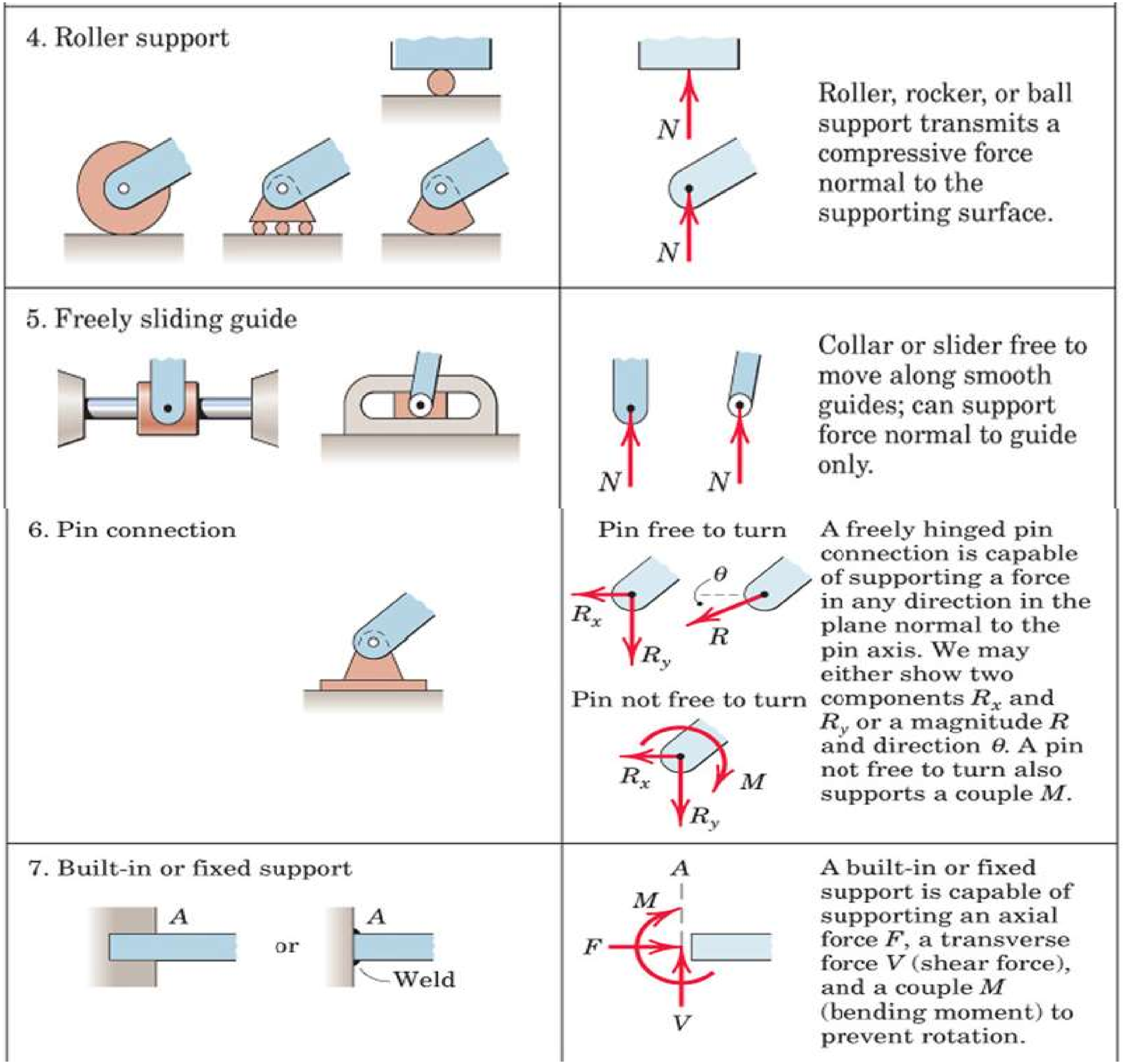
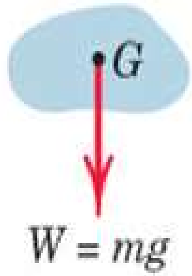
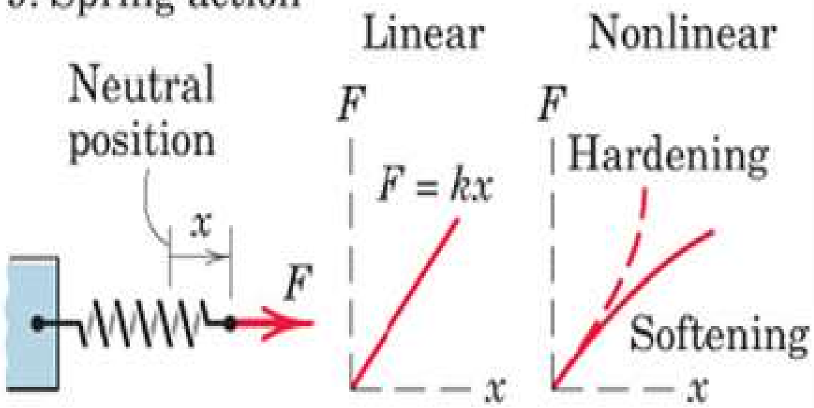

4. Roller support
N
N
Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide
N
N
Collar or slider free to move along smooth guides; can support force normal to guide only.
6. Pin connection
Pin free to turn
θ
R
Pin not free to turn
M
A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support
A
or
A
Weld
A
M
F
V
A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

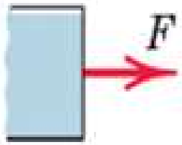
8. Gravitational attraction



The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.

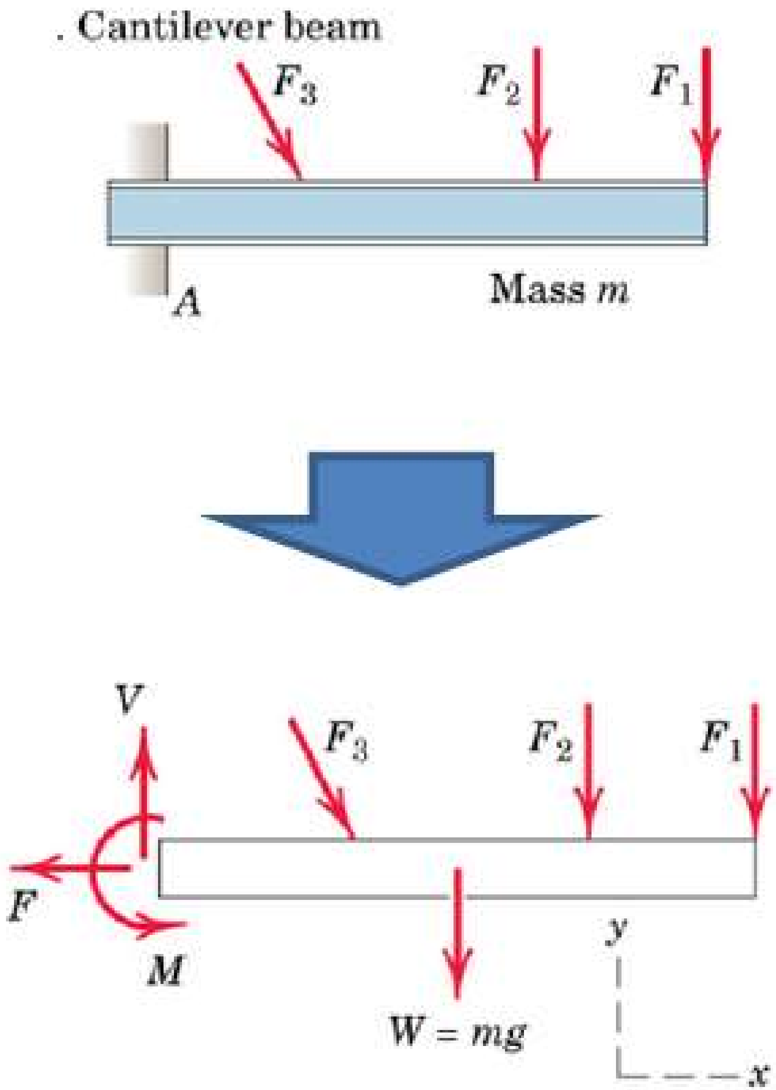
9. Spring action



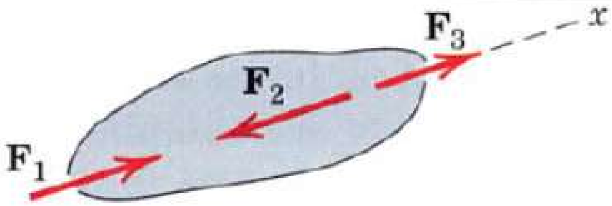
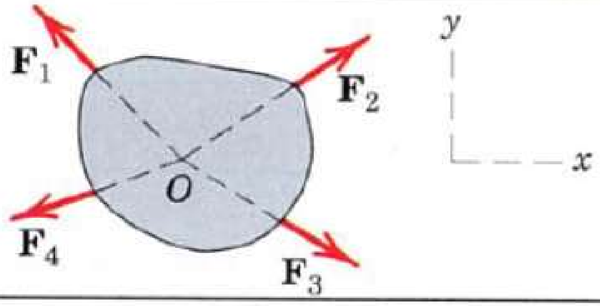
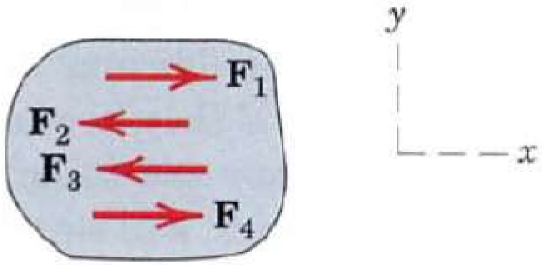
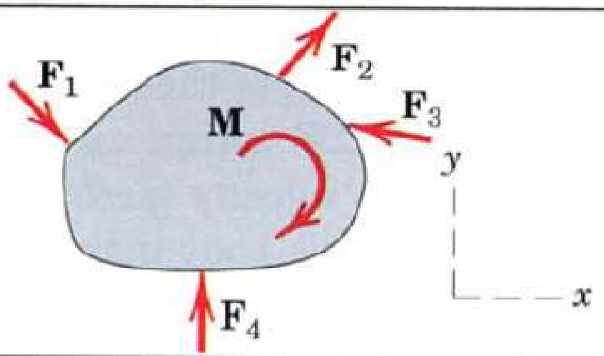


Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

- Free body diagram



CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS

Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\quad \Sigma M_z = 0$ $\Sigma F_y = 0$

- Types of Beams
- Cantilever
- Simply Supported Beam
- Continuous Beam

- Types of Loads
- Concentrated loads - W
- Uniformly distributed Loads – w.x at centroid
- Uniformly varying load – area enclosed by loading patern

- Reactions at supports can be determined by considering the system of forces including applied forces and reactions in equilibrium.

- Use eqns of equilibirium

12.2. TYPES OF LOADING

Though there are many types of loading, yet the following are important from the subject point of view :

1. Concentrated or point load,
2. Uniformly distributed load,
3. Uniformly varying load.

12.3. CONCENTRATED OR POINT LOAD

A load, acting at a point on a beam is known as a *concentrated or a point load* as shown in Fig. 12.1.

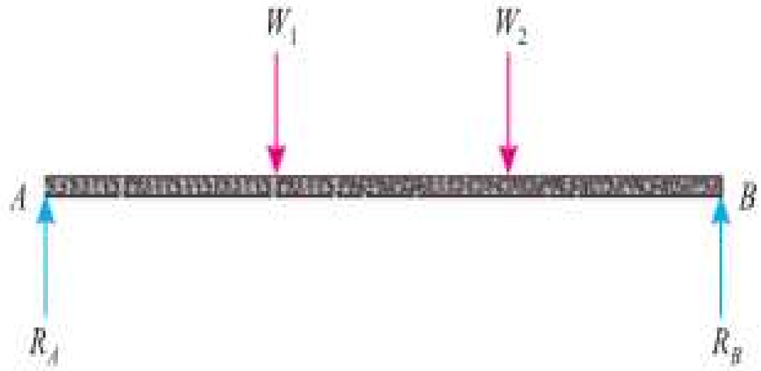


Fig. 12.1. Concentrated load.

In actual practice, it is not possible to apply a load at a point (*i.e.*, at a mathematical point), as it must have some contact area. But this area being so small, in comparison with the length of the beam, is negligible.

12.4. UNIFORMLY DISTRIBUTED LOAD

A load, which is spread over a beam, in such a manner that each unit length is loaded to the same extent, is known as *uniformly distributed load* (briefly written as U.D.L.) as shown in Fig. 12.2

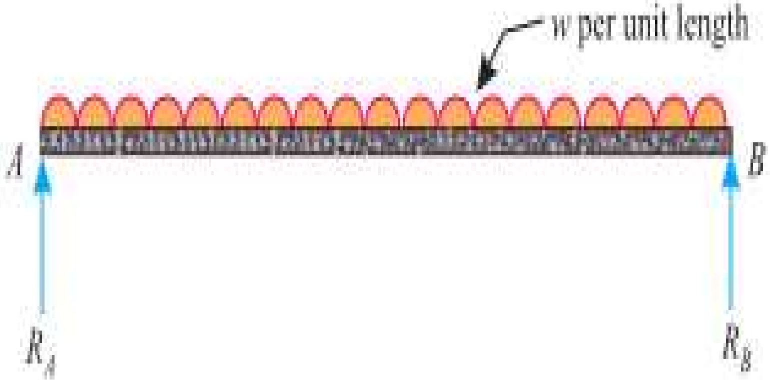


Fig. 12.2. Uniformly distributed load.

The total uniformly distributed load is assumed to act at the centre of gravity of the load for all sorts of calculations.

12.5. UNIFORMLY VARYING LOAD

A load, which is spread over a beam, in such a manner that its extent varies uniformly on each unit length (say from w_1 per unit length at one support to w_2 per unit length at the other support) is known as *uniformly varying load* as shown in Fig. 12.3.

Sometimes, the load varies from zero at one support to w at the other. Such a load is also called triangular load.

Fig. 12.3. Uniformly varying load.

Note : A beam may carry any one of the above-mentioned load system, or a combinations of the two or more.

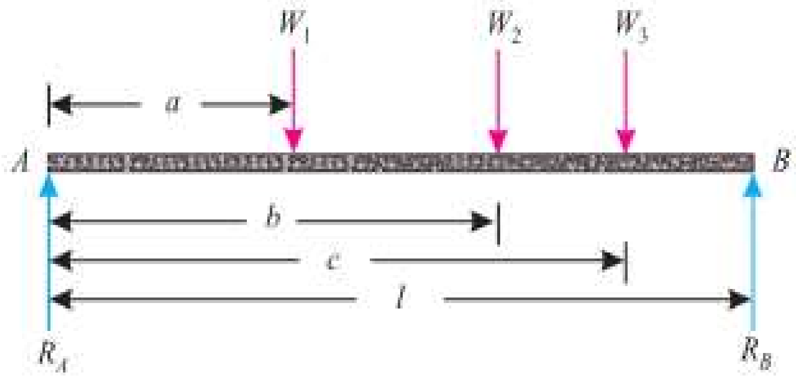


Fig. 12.4. Reactions of a beam.

Consider a *simply supported beam AB of span l, subjected to point loads W_1, W_2 and W_3 at distances of a, b and c, respectively from the support A, as shown in Fig. 12.4

Let R_A = Reaction at A, and

R_B = Reaction at B.

We know that sum of the clockwise moments due to loads about A

$$= W_1 a + W_2 b + W_3 c \qquad ...(i)$$

and anticlockwise moment due to reaction R_B about A

$$= R_B\, l \qquad ...(ii)$$

Now equating clockwise moments and anticlockwise moments about A,

$$R_B\, l = W_1\, a + W_2\, b + W_3\, c \qquad ...(\because \Sigma M = 0)$$

or

$$R_B = \frac{W_1 a + W_2 b + W_3 c}{l} \qquad ..(iii)$$

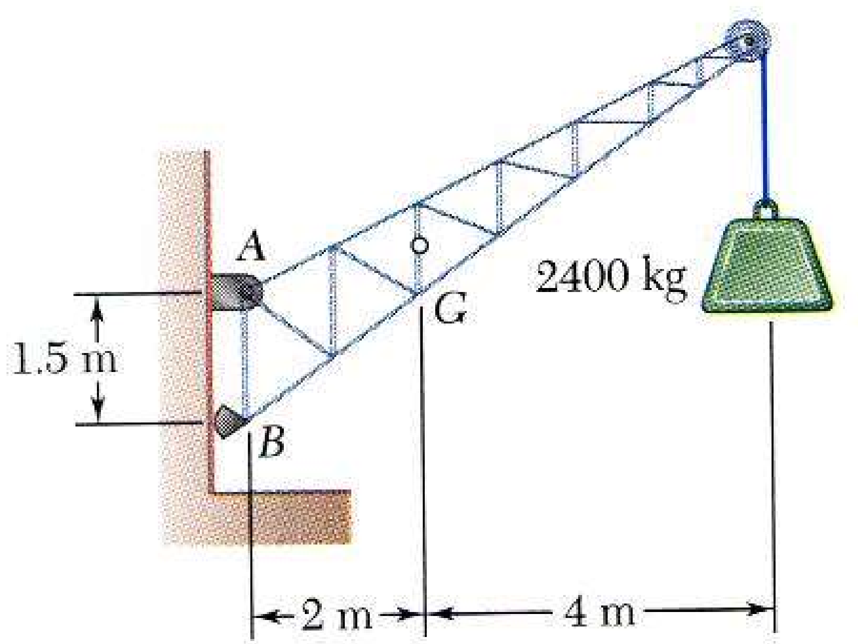
Since the beam is in equilibrium, therefore

$$R_A + R_B = W_1 + W_2 + W_3 \qquad ...(\because \Sigma V = 0)$$

and

$$R_A = (W_1 + W_2 + W_3) - R_B$$

- A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.
- Determine the components of the reactions at *A* and *B*.



- Determine *B* by solving the equation for the sum of the moments of all forces about *A*.

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

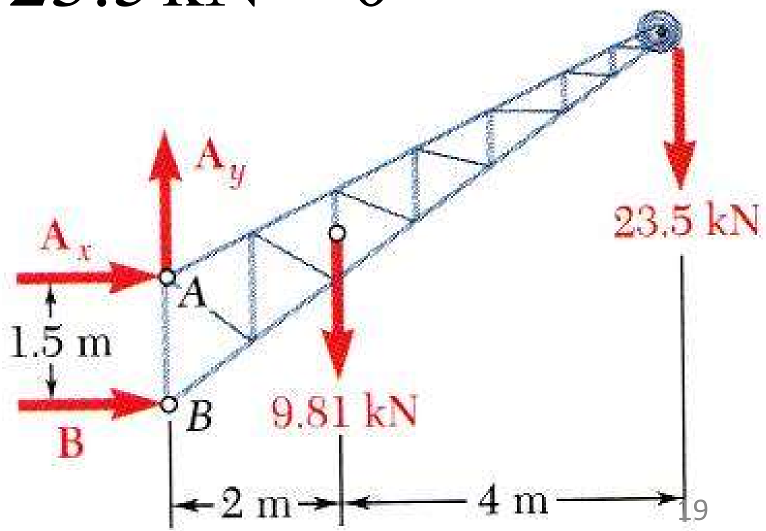
$$B = +107.1\,\text{kN}$$

- Determine the reactions at *A* by solving the equations for the sum of all horizontal forces and all vertical forces.

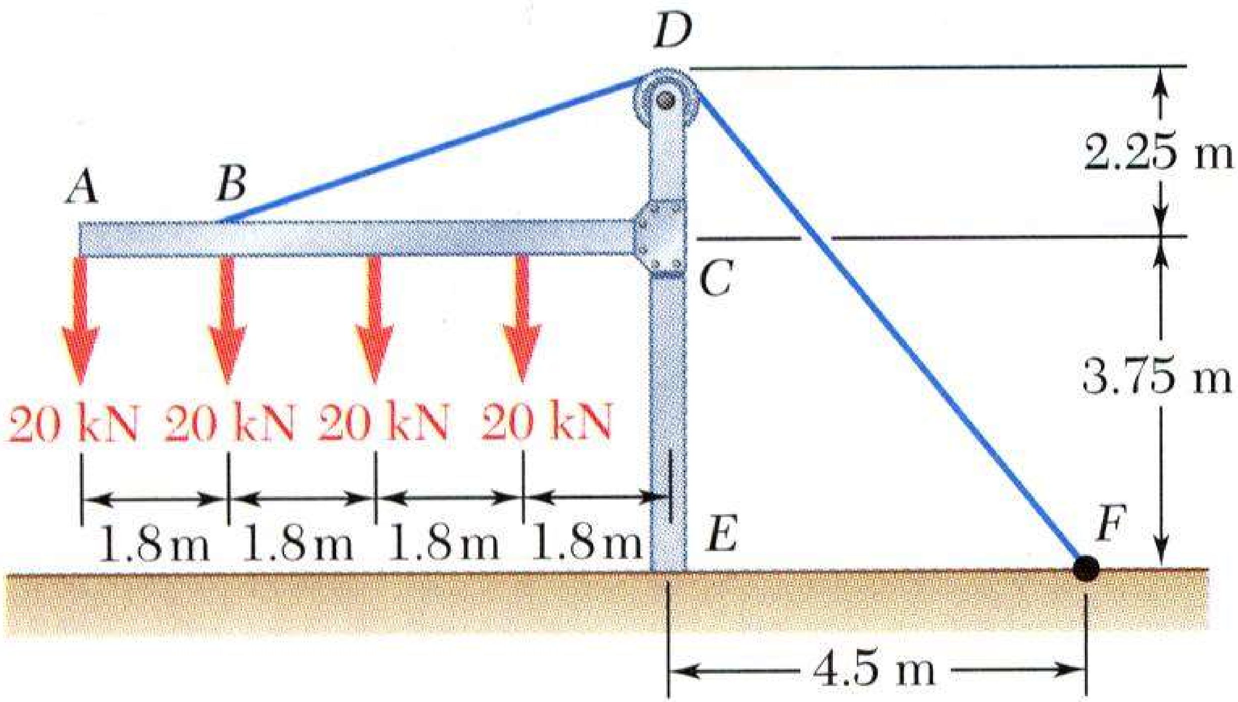
- $$\sum F_x = 0: \quad A_x + B = 0$$ $$A_x = -107.1\,\text{kN}$$

$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$A_y = +33.3\,\text{kN}$$



- The frame supports part of the roof of a small building. The tension in the cable is 150 kN. Determine the reaction at the fixed end *E*.



- Solve 3 equilibrium equations for the reaction force components and couple.

$$\sum F_x = 0: \quad E_x + \frac{4.5}{7.5}(150\,\text{kN}) = 0$$

$$\boxed{E_x = -90.0\,\text{kN}}$$

$$\sum F_y = 0: \quad E_y - 4(20\,\text{kN}) - \frac{6}{7.5}(150\,\text{kN}) = 0$$

$$\boxed{E_y = +200\,\text{kN}}$$

$$\sum M_E = 0: \quad +20\text{kN}(7.2\text{m}) + 20\text{kN}(5.4\text{m})$$

$$+20\text{kN}(3.6\text{m}) + 20\text{kN}(1.8\text{m})$$

$$-\frac{6}{7.5}(150\text{kN})4.5\,\text{m} + M_E = 0$$

$$\boxed{M_E = 180.0\,\text{kN}\cdot\text{m}}$$