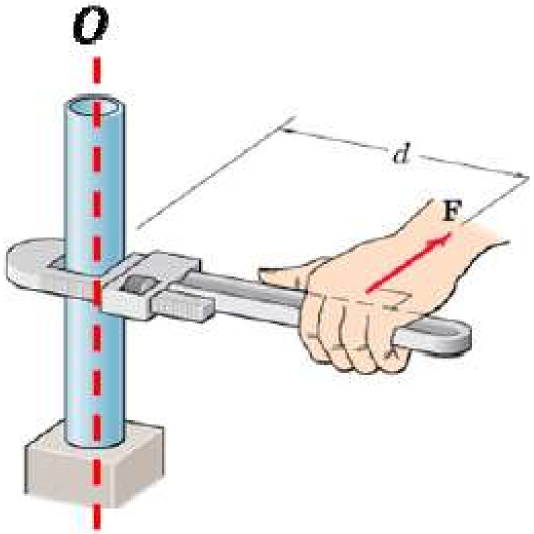
- **Moment of Force about an axis**
- The effectiveness of a force as regards to its tendency to produce rotation of a body about an axis or a point is called **moment of the force.**
- It is measured by the product of the magnitude of the force and arm of the force.
- **Arm of force**, or **moment arm** is the perpendicular distance of the line of action of the force from the moment centre.
- **Moment centre** is the point about which moment of a force is measured.

- **Moment of Force about an axis ...**
- The line perpendicular to the plane containing the force and passing through the moment centre is called **axis of the moment.**
- Moment of force will be expressed in (Nm).
- Vector qty
- Depending upon the relative position of the force and moment centre, the moment of a force will be either clockwise or counter clockwise.
- The sense of moment of a force is taken as clockwise when the force rotates or tends to rotate the arm of force in clockwise direction about the moment centre.

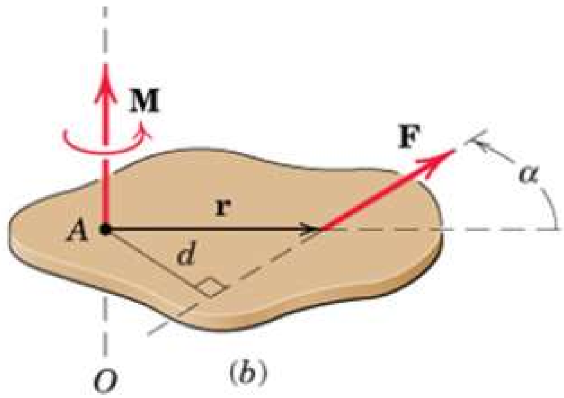


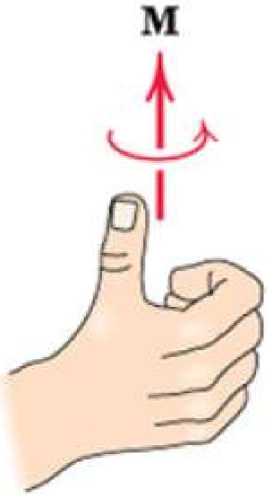
Moment about axis O-O is $\boldsymbol{M_o} = Fd$

Magnitude of $\boldsymbol{M_o}$ measures tendency of **F** to cause rotation of the body about an axis along $\boldsymbol{M_o}$.

Moment about axis O-O is $\boldsymbol{M_o} = Fr\sin\alpha$

$$\boldsymbol{M_o} = \boldsymbol{r} \times \boldsymbol{F}$$





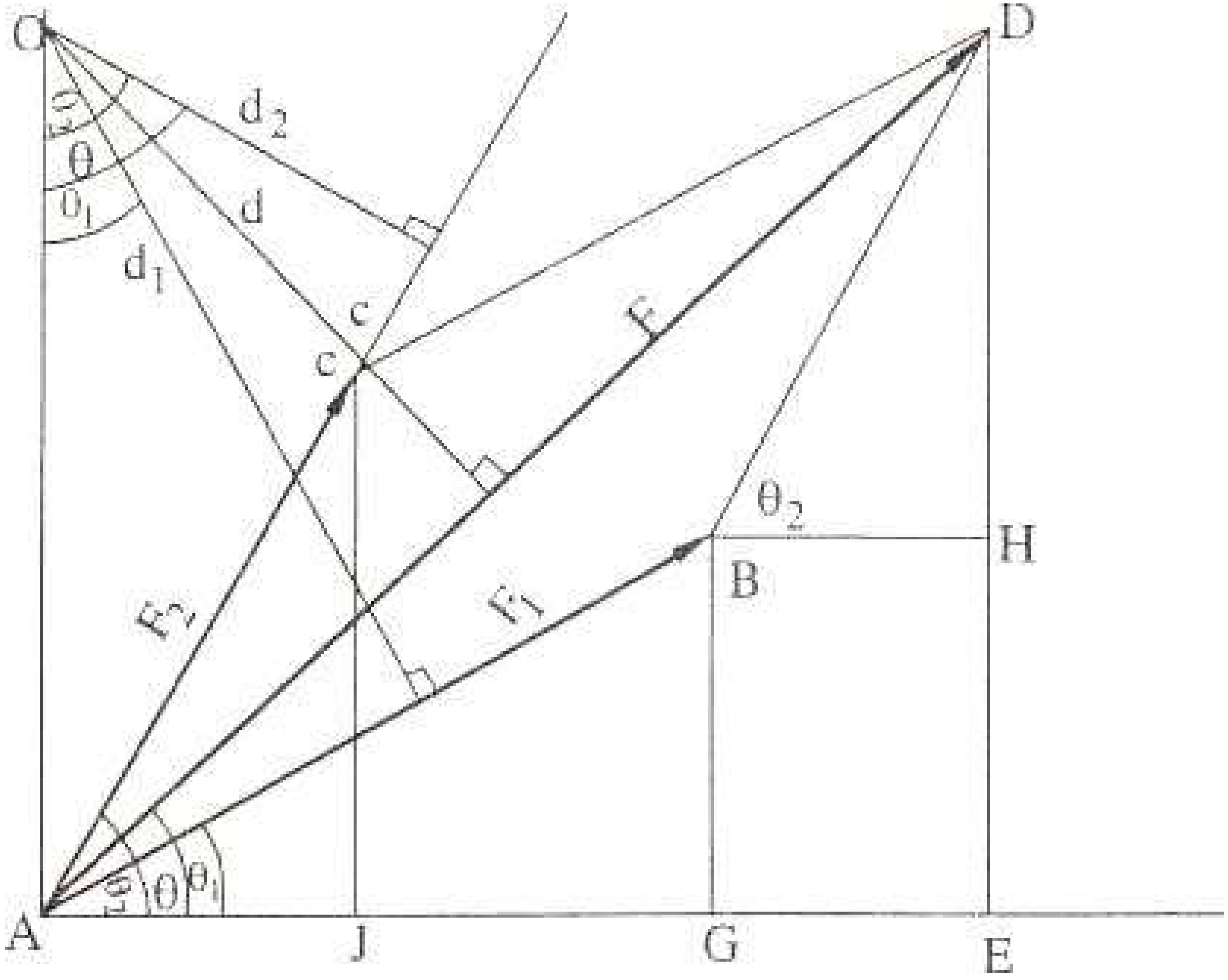
Sense of the moment may be determined by the right-hand rule

- Moment of a force will be maximum when the line of action of the force is perpendicular to the line joining the moment centre and point of application of the force.

- Moment of a force will be zero when,
  - (i) the force acts at the moment centre itself and
  - (ii) when the line of action of the force passes through the moment centre.

- **Varignon’s theorem of moments. - principle of moments**

- French mathematician Varignon (1654-1722)
-
- states that the moment of a force about any axis is equal to the sum of moments of its components about that axis.



O
D
d_2
θ_3
θ
θ_1
d
d_1
c
c
F
θ_2
H
B
F_2
F_1
θ_3
θ
θ_1
A
J
G
E

- Consider a force F acting at a point A. F_1 and F_2 are the components of F along any two directions.
- The moment of F about an axis through an arbitrary point O is F x d, where d is the arm of force F.
- d_1 and d_2 are the arm of forces of F1 and F2, respectively.
- Sum of moments of the components F1 and F2, about O ; F1. d1 + F2. d2
- It is to be proved that **$F \times d = F_1 \times d_1, + F_2 \times d_2$.**

- $AE = AG + GE$; $= AG + BH$; $= AG + BD \cos \theta_2$
- $= AG + AC \cos \theta_2$
- $AD \cos \theta = AB \cos \theta_1 + AC \cos \theta_2$.
- $F \cos \theta = F_1 \cos \theta_1 + F_2 \cos \theta_2$.
- Multiplying by OA.
- $F \times OA \cos \theta = F_1 \times OA \cos \theta_1 + F_2 \times OA \cos \theta_2$
- $F \times d = F_1 \times d_1 + F_2 \times d_2$

- **Moment of F about O = moment of F_1 about O + moment of F_2 about O.**
- **Moment of given force about any point = Sum of moments of its components about the same point**

- Calculate the moment of the given force F = 10 kN about point O

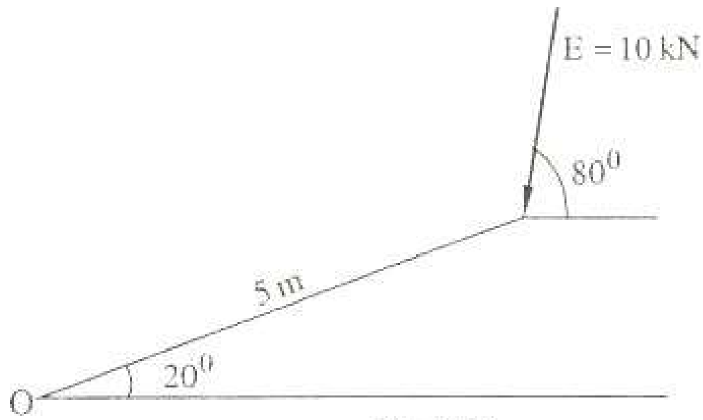
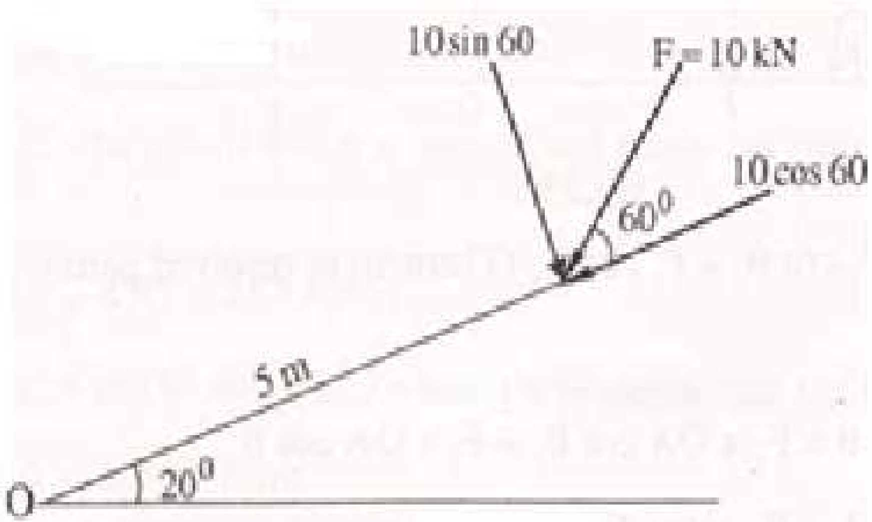
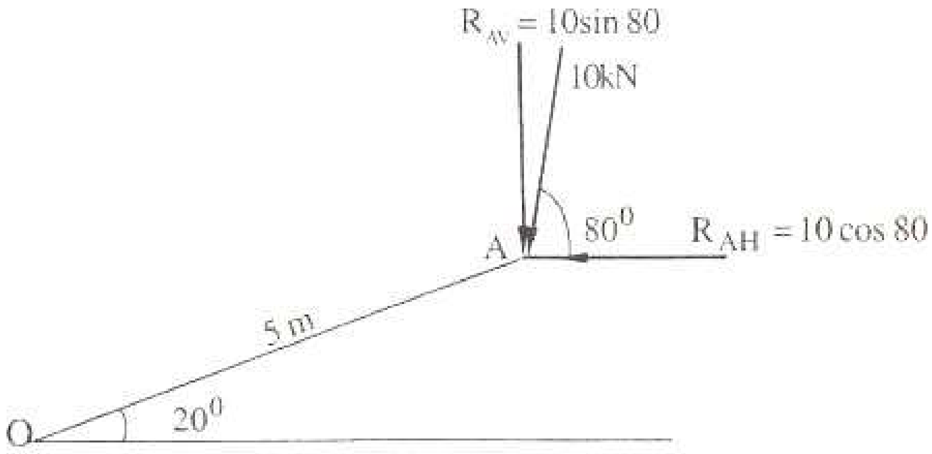


Fig.2.97

- **Resolving along and perpendicular to incline plane**
- M_o = (10 cos 60) x 0 + (10 sin 60) x 5 = 43.30 kNm.
- **Resolving vertical and horizontally**
- M_o= 10 sin 80 x5 cos 20- 10 cos 80 x 5 sin 20
- = 43.30 kNm.





- **Coplanar non concurrent forces**

- **Resultant of coplanar non concurrent forces.**

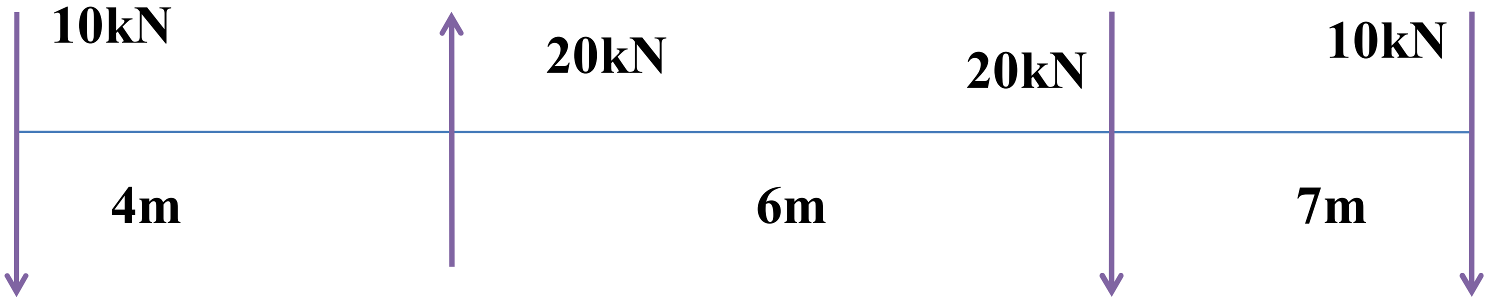
- Magnitude and direction of resultant of a number of coplanar concurrent forces, can be obtained by resolving the forces along two mutually perpendicular directions and then finding the resultant of the sum of components of the forces.
- Resultant will be acting at the point of intersection of the concurrent forces.
- Magnitude and direction of non concurrent forces can also be obtained by resolving the forces along two perpendicular directions.

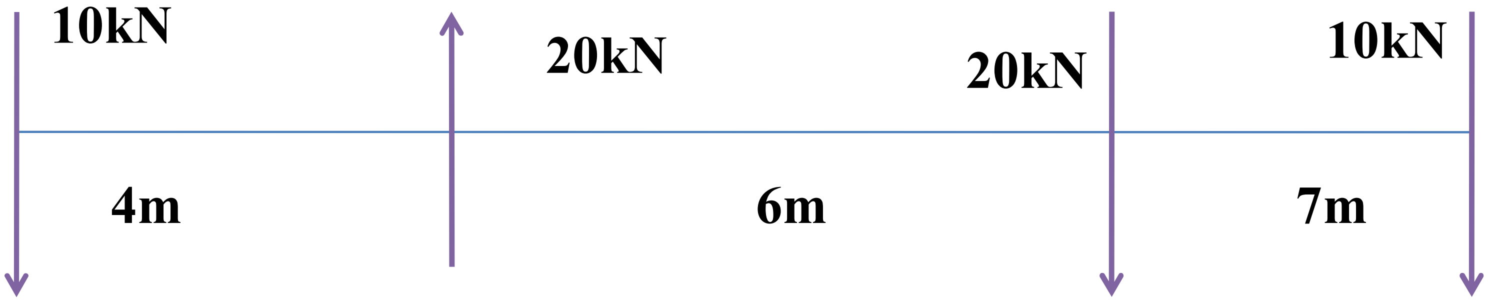
- **Resultant of coplanar non concurrent forces…**

- To obtain the position of resultant force, equate the sum of moments off all the forces in the plane, about any point in the plane, with the moment of their resultant about the same point.

- If the sum of moments of all the forces about a point is clockwise, then the position of resultant with respect to the point should be such that the moment of resultant about that point must also be clockwise.

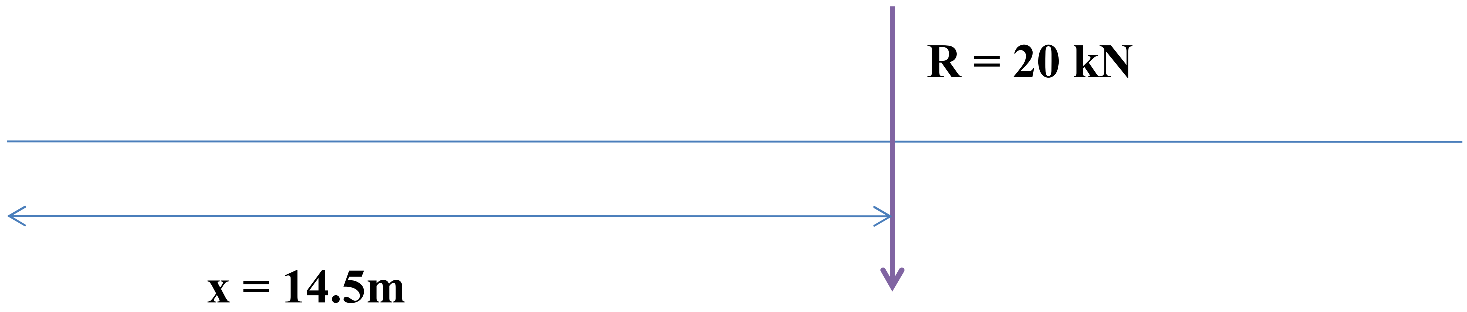
- Determine the resultant of the system of forces



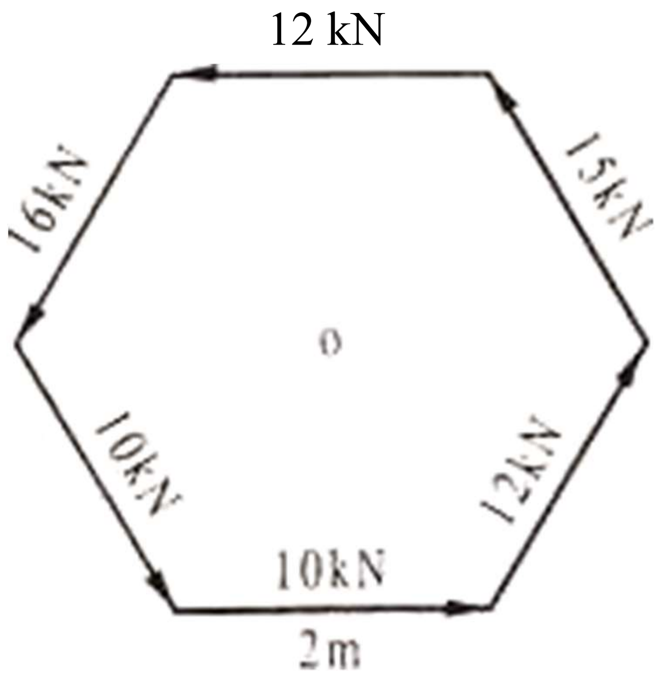


- Since there is no horizontal or inclined force,
- Σ Fh = 0;
- Σ Fv = 0; -10+20 -20 -10 = -20 kN
- Resultant R = $[(\Sigma Fx)^2 + (\Sigma Fy)^2]^{1/2}$
- R = 20 kN ; $\theta = 90$
- Inclination of resultant with positive X axis,
- $\theta_R = 180 + \theta = 270°$

- Let the distance of line of action of this force from end A be x,
- then $\Sigma M_A = R. x$
- $\Sigma M_A = 10 \times 0 - 20 \times 4 + 20 \times 10 + 10 \times 17$
- $= - 80 + 140 + 170 = 290$ kNm c.w.
- Moment of R about A should be clockwise,
  - (for this R must be towards right of A)
- $\Sigma M_A = R . x$
- $x = 230/20 = 14.5$ m



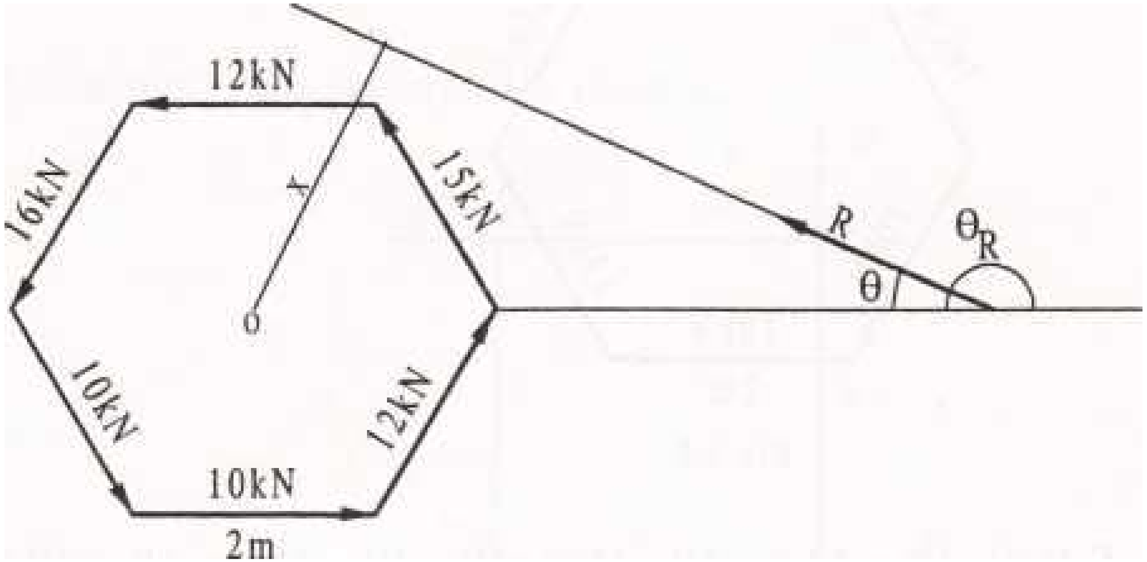
- Six forces of magnitude 10 kN, 12 kN, 15 kN, 12 kN, 16 kN and 10 kN are acting along the sides of a regular hexagon of side 2 m in order.
- Find the resultant force and its direction.
- Find also the position of the resultant with respect to the centre of the hexagon.



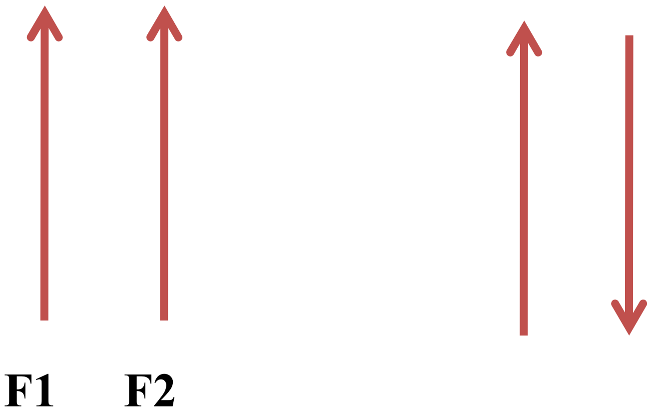
- Σ Fh = 10 + 12 cos 60 -15cos 60 -12- 16cos60 + 10cos60 = - 6.5 kN
- Σ Fv= 0+12 sin 60 + 15 sin 60 + 0 + 16 sin 60 -10 sin 60 = 0.866 kN
- Resultant force R = 6.56 kN
- θ = 7.59
- Inclination of resultant θ_R=180-θ= 180 - 7.59= 172.41
- Sum of moments of all the forces about the centre of hexagon,
- Σ M = = - (10 + 12 + 15 + 12 + 16 + 10) x 2 sin 60

  = - 75 x 2 sin 60 = 129.90 (ccw)

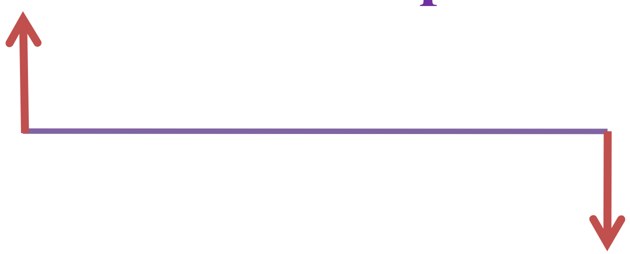
- Let the distance of resultant force from the centre be x.
- moment of resultant about the centre = R .x,
- Equating these two moments,
- R . x = 129 .90
- X = 19.80 m



- **Parallel forces and couples.**
- forces whose lines of action are parallel to each other - parallel forces.
- **Like parallel and unlike parallel forces**



- **Clockwise couple & anticlockwise couple**

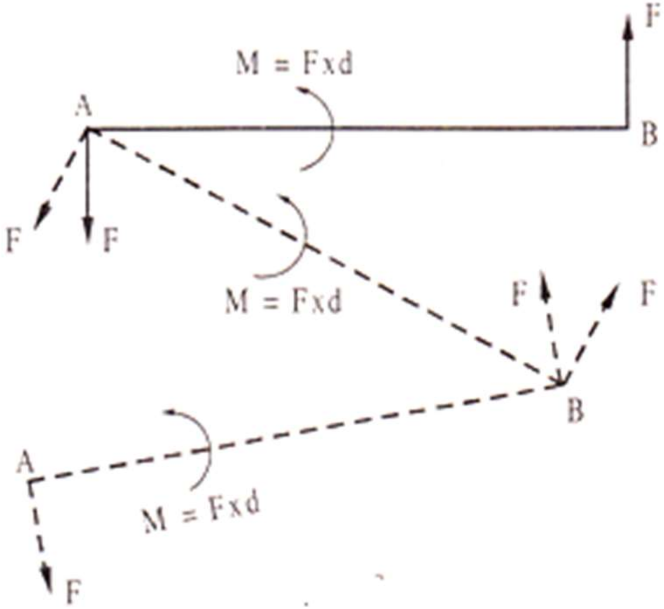


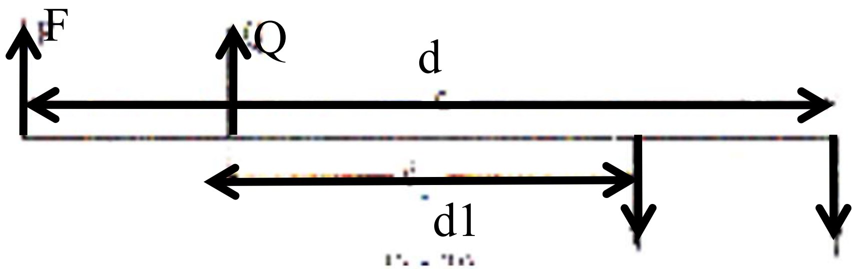
- The resultant of two unlike parallel forces is zero. Hence these forces cannot be replaced by another single force.
- Such two forces having the same magnitude, parallel line of action, and opposite sense are said to form a **couple.**
- The plane in which the forces act is called the **plane of the couple**.
- The distance between the line of action of forces is **arm of couple.**
- moment of a couple = product of the magnitude of one of the forces and the arm of couple.

- **Properties of force couple**

- two forces constituting a couple are equal in magnitude and opposite in direction. Therefore, the sum of forces of a couple is zero.
- the action of a couple on a rigid body will not be changed if its arm is turned in the plane of couple through any angle about one of its ends
- without changing the action on a body; a given couple can be replaced by another one with different forces with a different arm, provided the moments of the two couples are equal.
- Several couples in one plane can be replaced by a single couple acting in the same plane such that the moment of this single couple is equal to the algebraic sum of the moments of the given couples.

- # Properties of force couple…

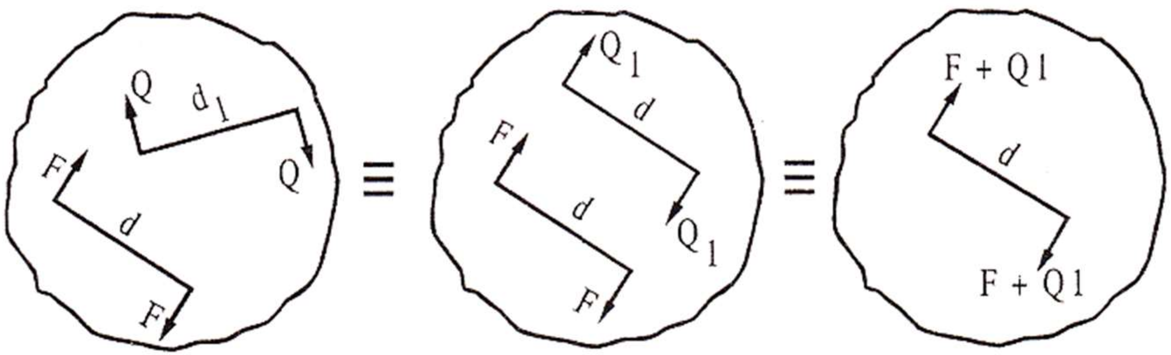




$F \times d = Q \times d1$

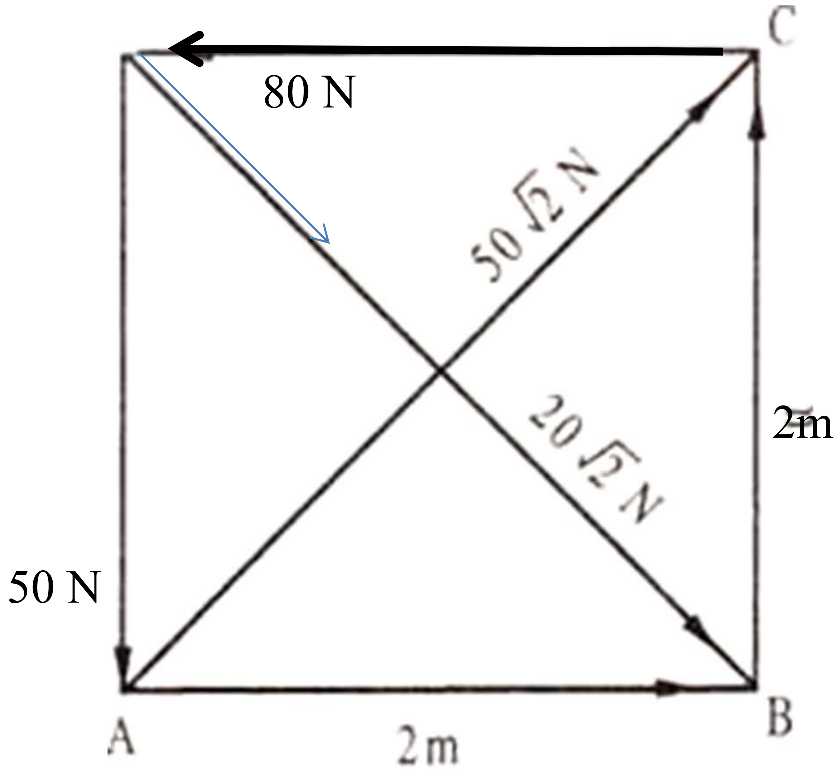
$\mathbf{F \times d + Q\, d_1} = F \times d + Q_1 \times d$

$= \mathbf{(F + Q_1)\, d}$ where Q_1 is $Q1 \times d = Q \times d1$



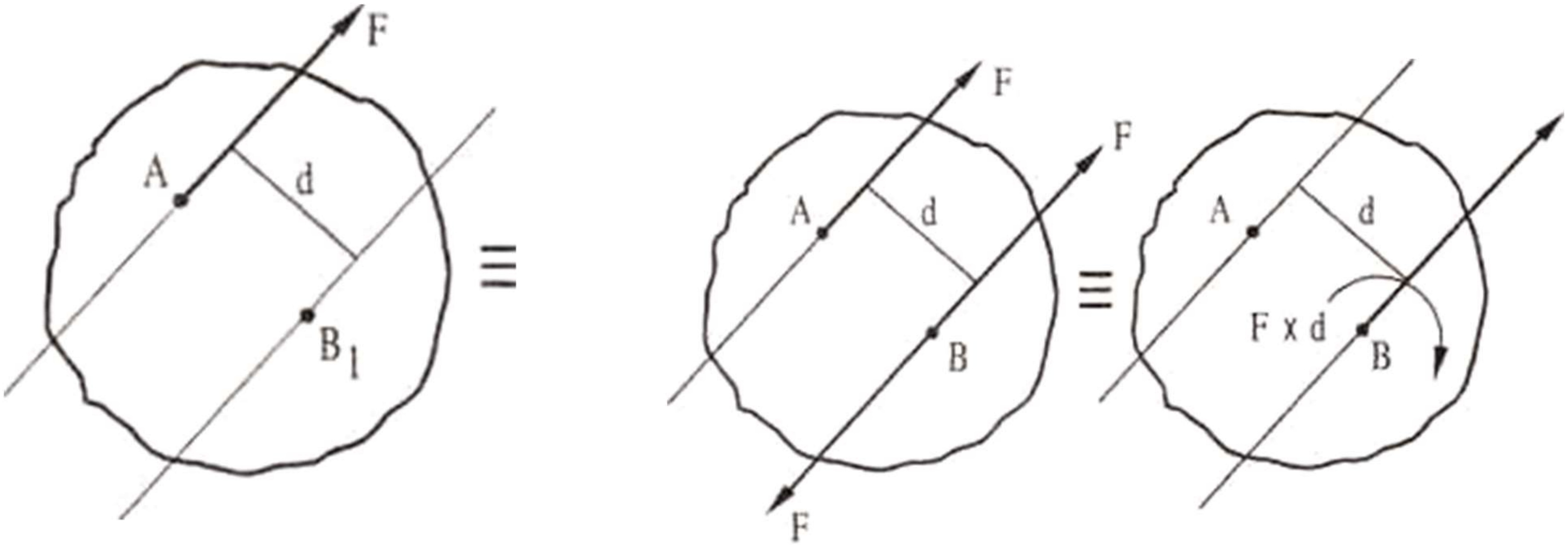
$Q_1 = Q\, d_1 / d$

- ABCD is a square whose side length is 2 m. Forces of magnitude 10,20,80 and 50 N act along AB, BC, CD and DA respectively. Forces of magnitude $50\sqrt{2}$ N and $20\sqrt{2}$ N act along the diagonal AC and DB respectively. Show that they are equivalent to a couple and calculate the moment of this couple.



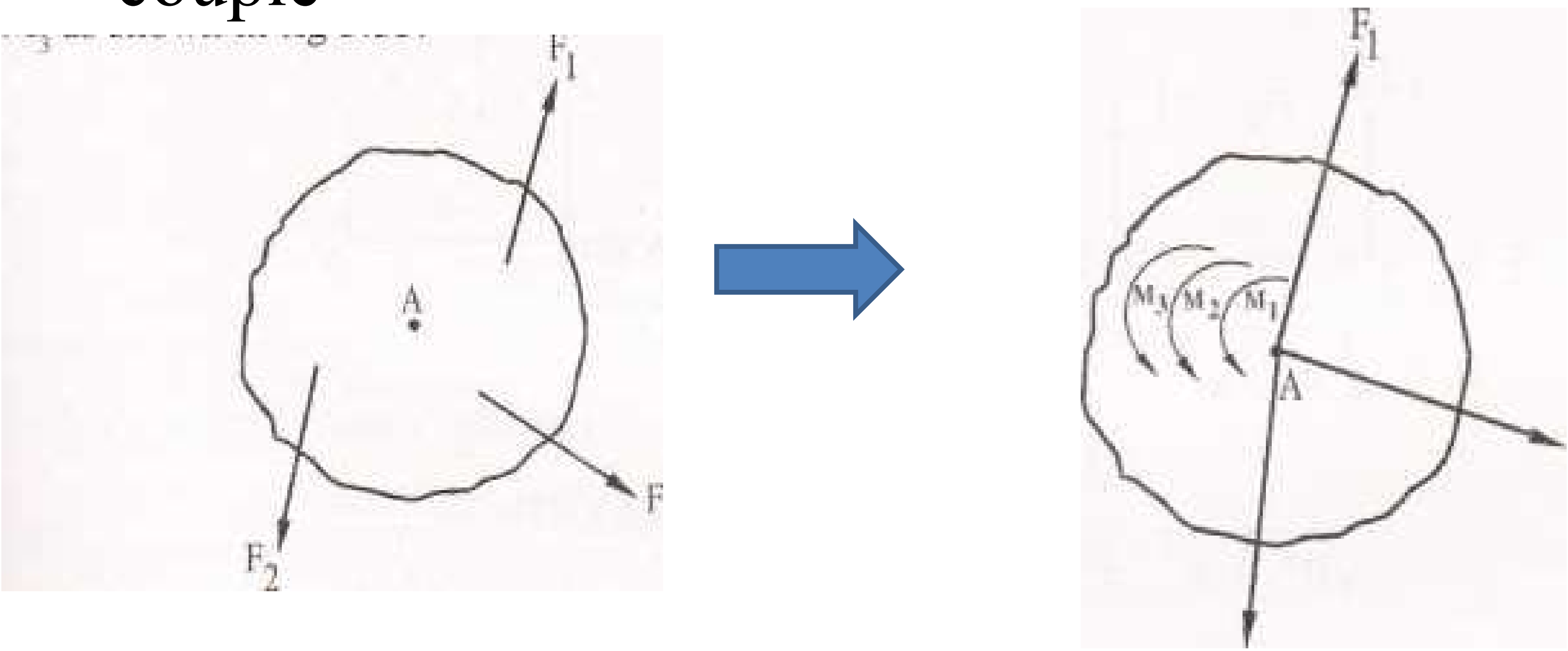
- Resolving the forces horizontally,
- $\Sigma F_h = 10 - 80 + 50\sqrt{2} \times \cos 45 + 20\sqrt{2} \times \cos 45$
- $= 0$
- lly $\Sigma F_v = 0$
- Resultant R= 0
- Taking moments of forces about A,
- $\Sigma M_A = (20\sqrt{2} \cos 45) \times 2 - 80 \times 2 - 20 \times 2$
- $= 40 - 160 - 40$
- $= -160$ N m = 160 N m c.c.w.
- **Since the resultant force is zero and the moment is not equal to zero, the system is equivalent to a couple of moment 160 Nm c.c.w.**

- **Resolution of a given force into force acting at a given point and a couple.**

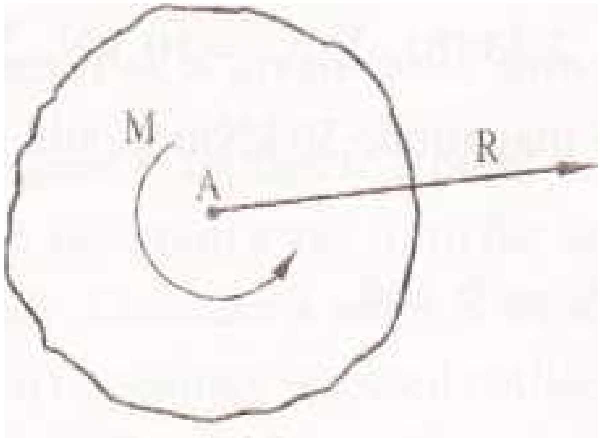


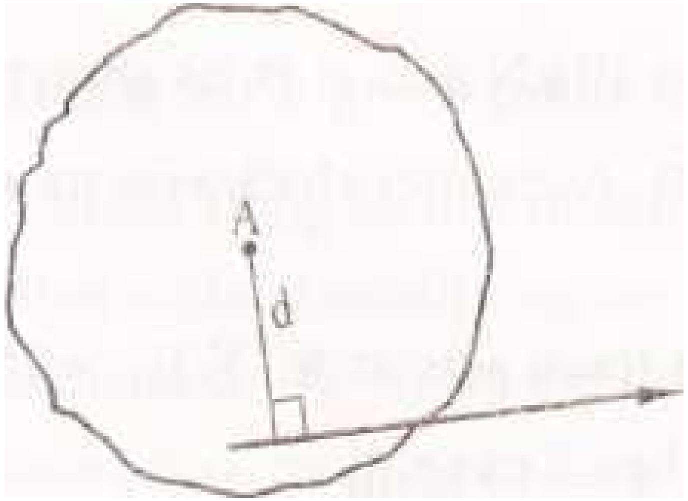
- force F acting at A can be resolved into a force acting at B together with a couple of magnitude F x d

- Reduction of system of coplanar forces acting on a rigid body into a single force and single couple



- Reduction of system of coplanar forces acting on a rigid body into a single force and single couple…..



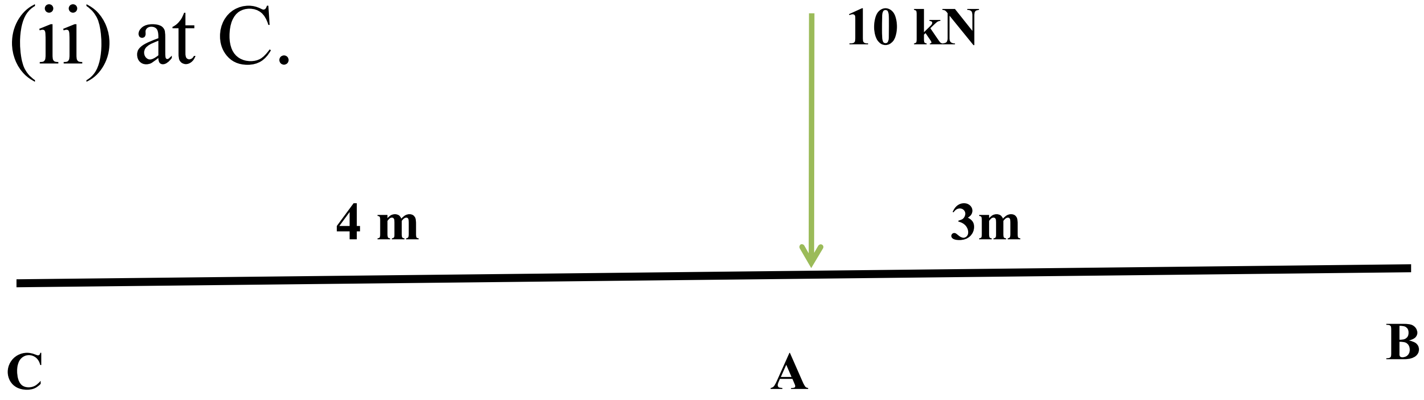


At support O

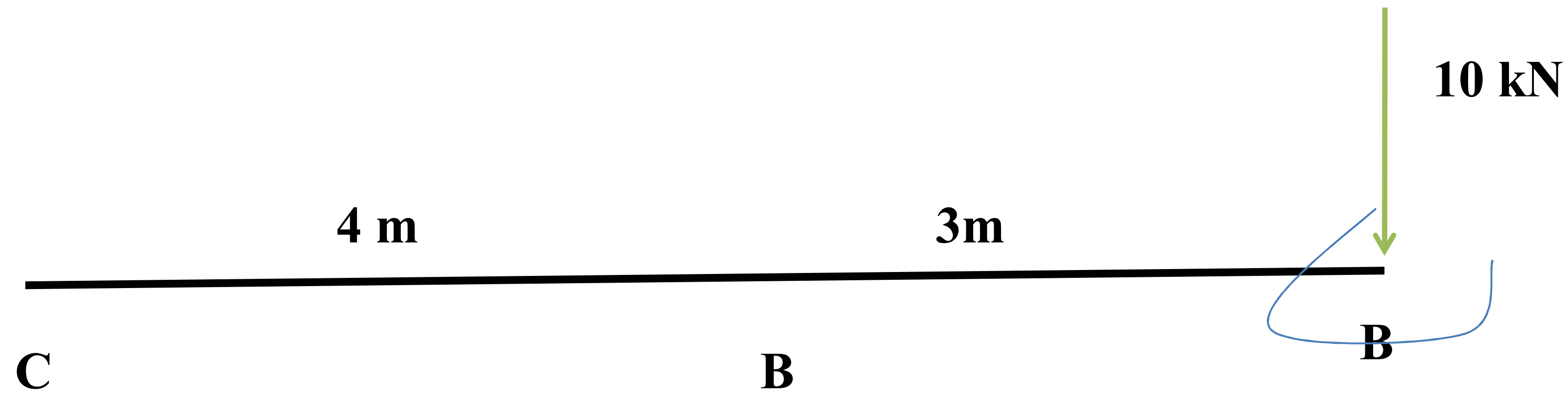
$$W_r = W_1 + W_2$$

$$M_o = W_1 d_1 + W_2 d_2$$

- Replace the force acting at A 10 kN by a force and couple at
- (i) B and
- (ii) at C.

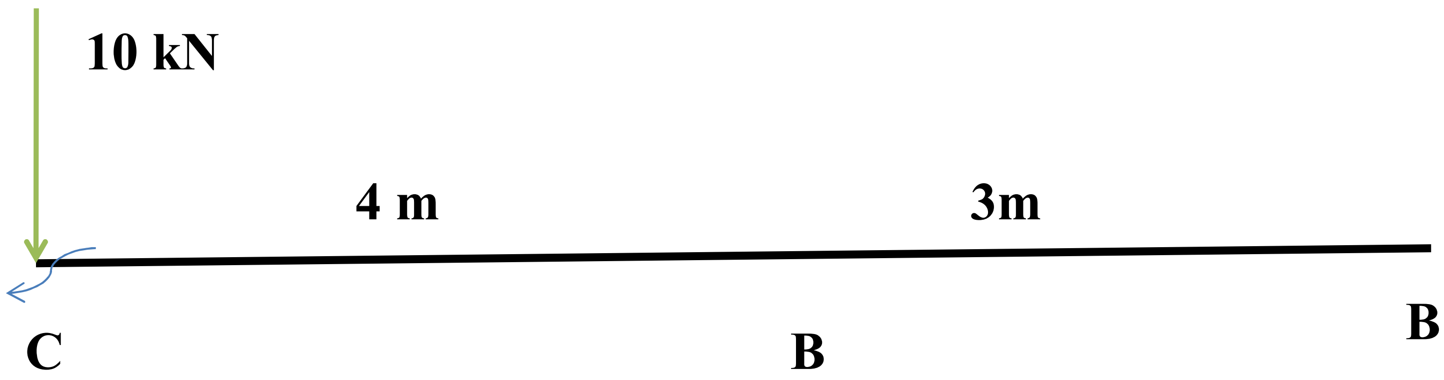


- Moment at B = -10 * 3 = 30 kNm
- When force acts at B, ΣFv = 10 and ΣMb =0



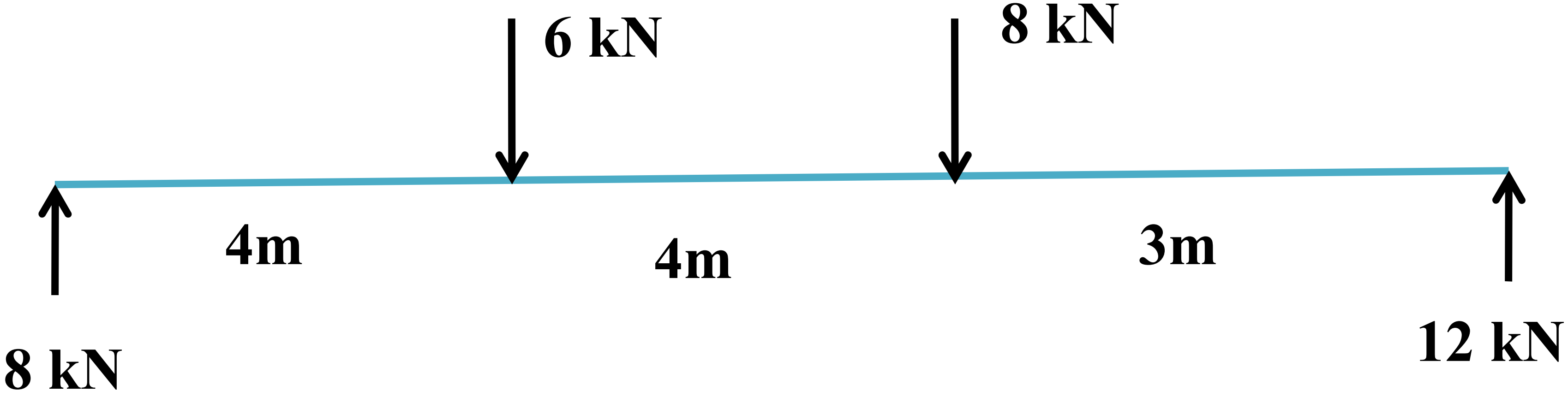
- So a counter clockwise moment have to be applied at B 30kNm

- Moment at C = 10 * 4 = 40 kNm
- When force acts at C, ΣFv = 10 and ΣMC =0

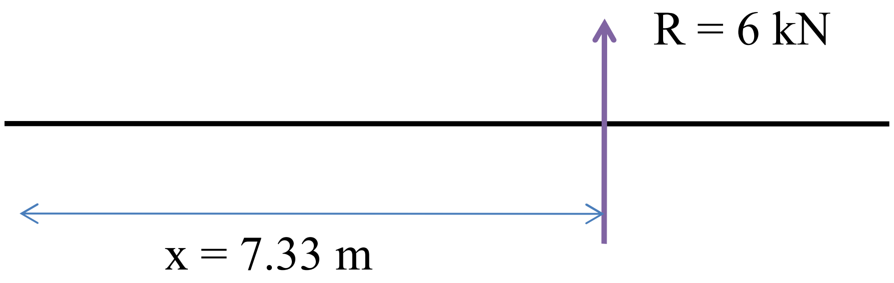


- So a clockwise moment have to be applied at C 40kNm

- A rigid bar AB is acted upon by forces as shown in fig.
- Reduce the force system to
- (i) a single force
- (ii) force moment system at A, and
- (iii) force moment system at D.



- $\Sigma Fv = 6$ kN
- R = 6 kN
- Case-1
- Let resultant be at a distance of x from A
- $\Sigma Ma = 6 \times 4 + 8 \times 8 - 12 \times 11 = -44$ kNm ccw
- Moment of resultant about A = 6 . x
- 6x = 44
- x = 7.33 m

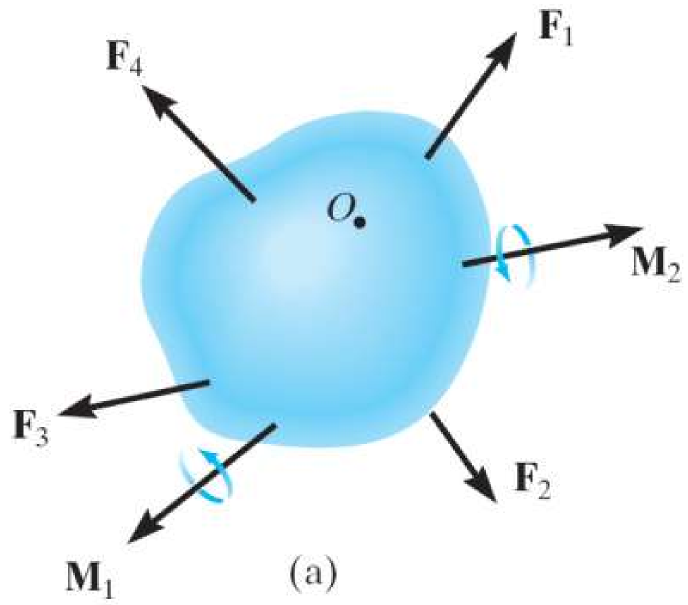


- Case -2 – force moment system at A
- Ma = 44 kNm ccw; resultant is 6 kN upwards

- Case -3 – force moment system at D
- Md = 4 kNm cw; resultant is 6 kN upwards

- **Rigid Body Equilibrium**
- A rigid body will remain in equilibrium provided
  - Sum of all the external forces acting on the body is equal to zero, and
  - Sum of the moments of the external forces about a point is equal to zero

$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$



(a)