Friction

- Friction – Characteristics of dry friction –
- Problems involving friction of ladder, wedges and connected bodies.

Friction: Definition

- Whenever the surface of two bodies are in contact with each other , there will be limited amount of resistance to sliding between them which is called friction

- Force of friction comes into play when two surfaces in contact with each other exert force normal to each other and one surface slides or tends to slide with respect to each other

- 2 types of friction:
  - **Coulomb or dry friction and**
  - **Fluid friction or wet friction**
  - Fluid friction applies to lubricated mechanisms.

Laws of friction

1. Force of friction always acts in a direction opposite to the direction in which the body moves or tends to move
2. The force of friction is equal to the force which tends to move the body up to limiting value (f_{max})
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between two contact surfaces
4. Force of friction depends up on the roughness of the surface in contact
5. Force of friction is independent of area of contact between two surfaces

6. For low velocities, frictional force is independent of magnitude of velocity. But generally the dynamic friction is less than the limiting friction

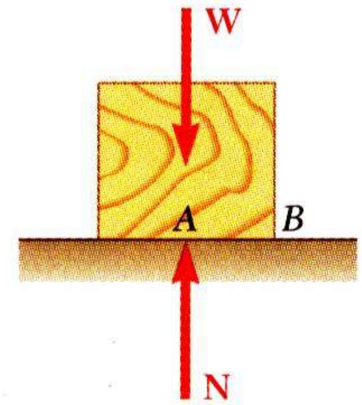
Coefficient of friction:

- The magnitude of limiting friction bears a constant ratio to the normal reaction between two surfaces
- $f \alpha R_N$
- Coefficient of friction at contact surface

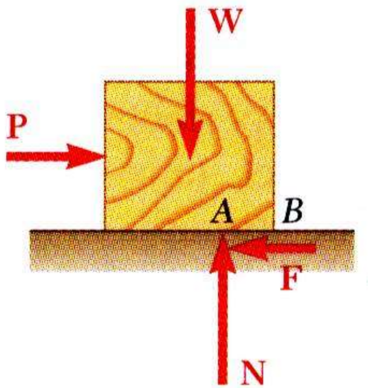
$$f = \mu R_N$$

$$\mu = \frac{f}{R_N}$$

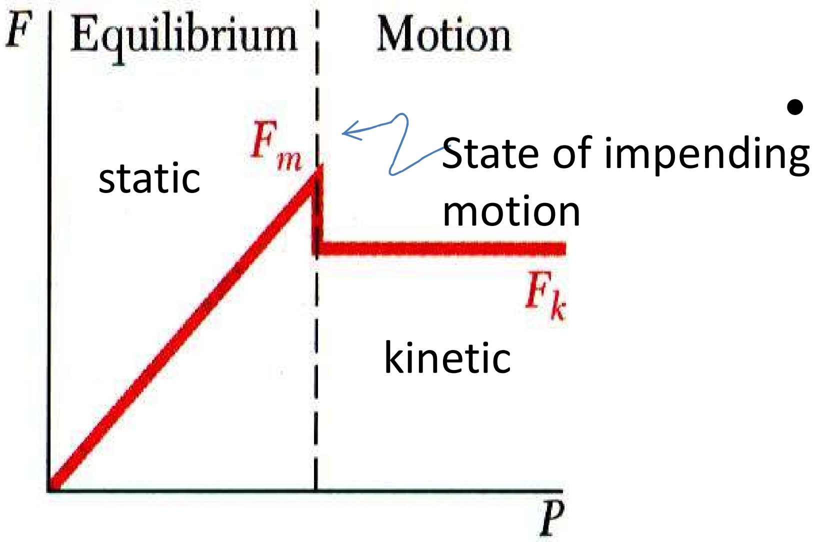
- The Laws of Dry Friction. Coefficients of Friction



- Block of weight *W* placed on horizontal surface. Forces acting on block are its weight and reaction of surface *N*.



- Small horizontal force *P* applied to block. For block to remain stationary, in equilibrium, a horizontal component *F* of the surface reaction is required. *F* is a *static-friction force*.



- As P increases, the static-friction force F increases as well until it reaches a maximum value F_m.

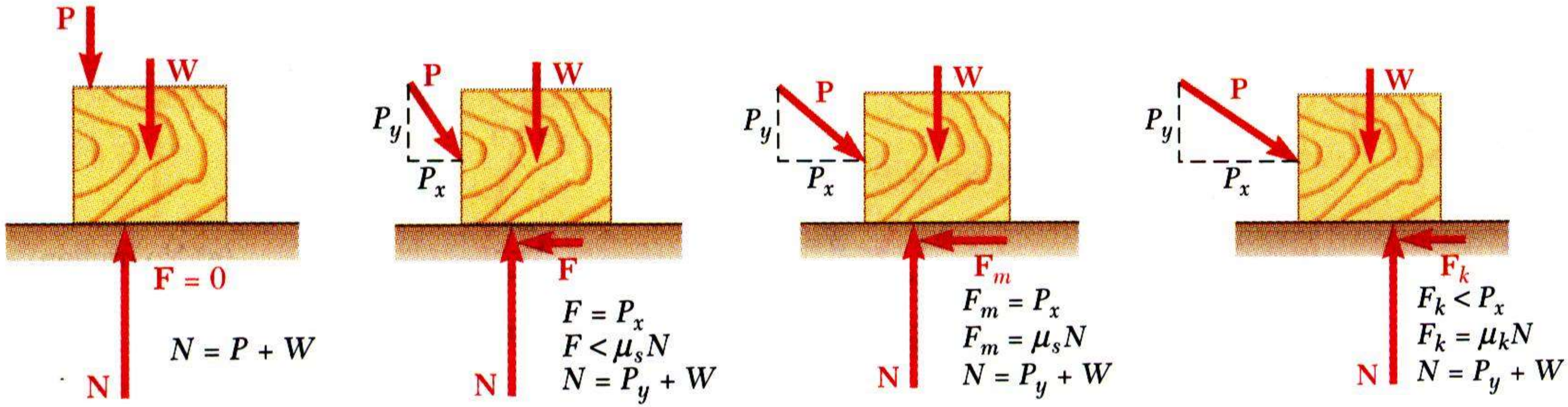
$$F_m = \mu_s N$$

- Further increase in P causes the block to begin to move as F drops to a smaller kinetic-friction force F_k.

$$F_k = \mu_k N$$

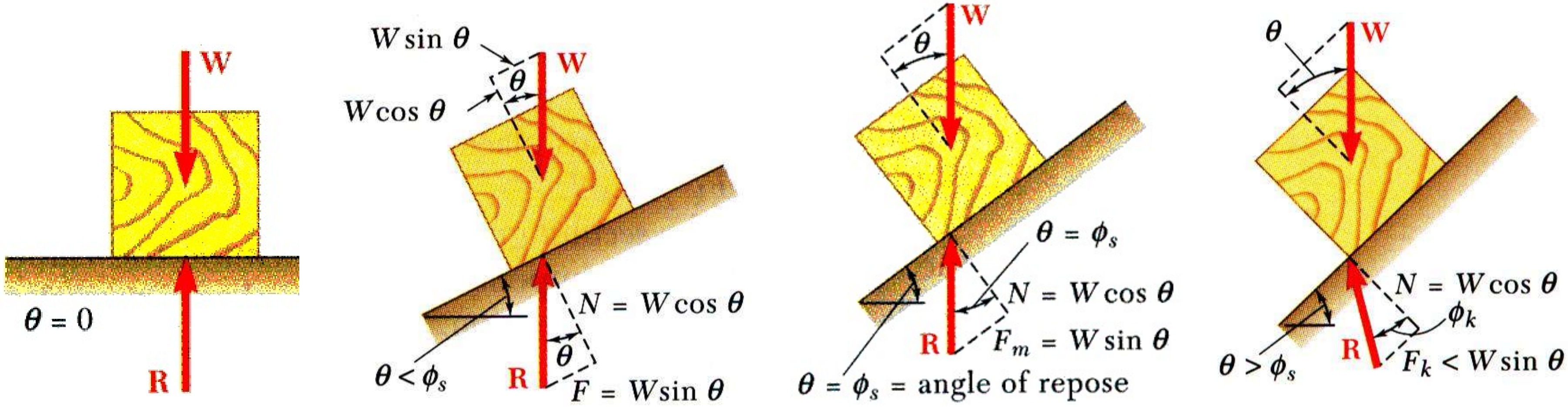
- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

- Four situations can occur when a rigid body is in contact with a horizontal surface:



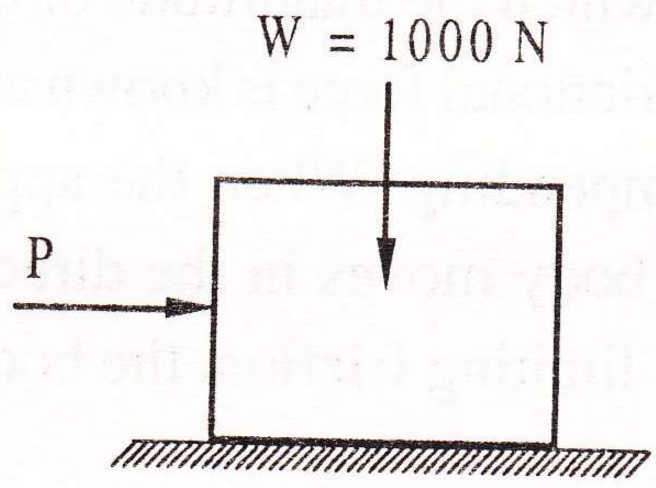
- No friction, ($P_x = 0$)
- No motion, ($P_x < F_m$)
- Motion impending, ($P_x = F_m$)
- Motion, ($P_x > F_m$)

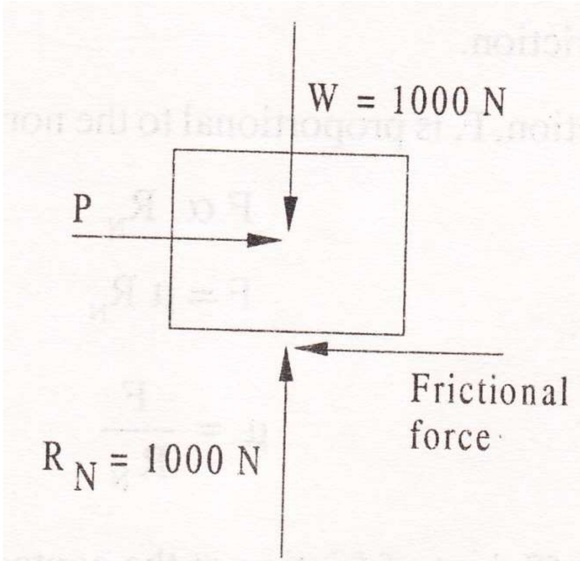
- Consider block of weight *W* resting on board with variable inclination angle θ.



- No friction
- No motion
- Motion impending
- Motion

- A body of weight 1000 N is kept on a horizontal rough surface. Coefficient of static friction is 0.2 and coefficient of dynamic friction is 0.18.
- Calculate the frictional force when the applied force P as shown in fig. is
  - (i) 0,
  - (ii) 100 N,
  - (iii) 200 N and
  - (iv) 300 N





- **When P = 0**

  The body has no tendency to move in the horizontal direction and hence the frictional force is 0

- **When P is 100 N**

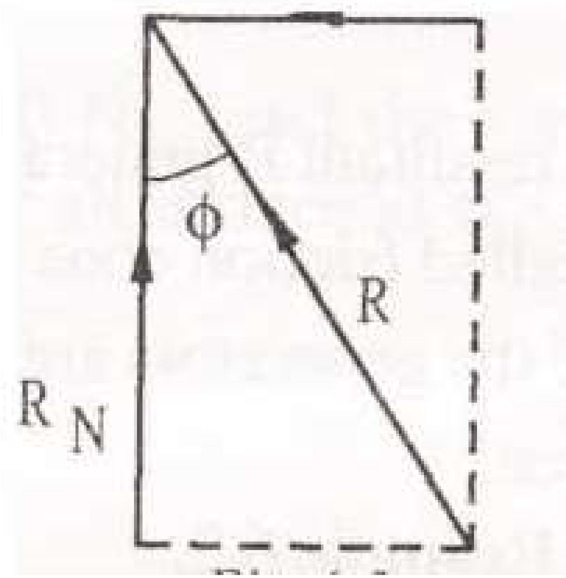
  $\mu = F / R_N$; $F = 0.2 \times 1000 = 200$ N

- Since the force tending the motion is less than the limiting friction the body does not move. But there is a tendency to move the body towards right.
- The frictional force = applied force = 100 N.

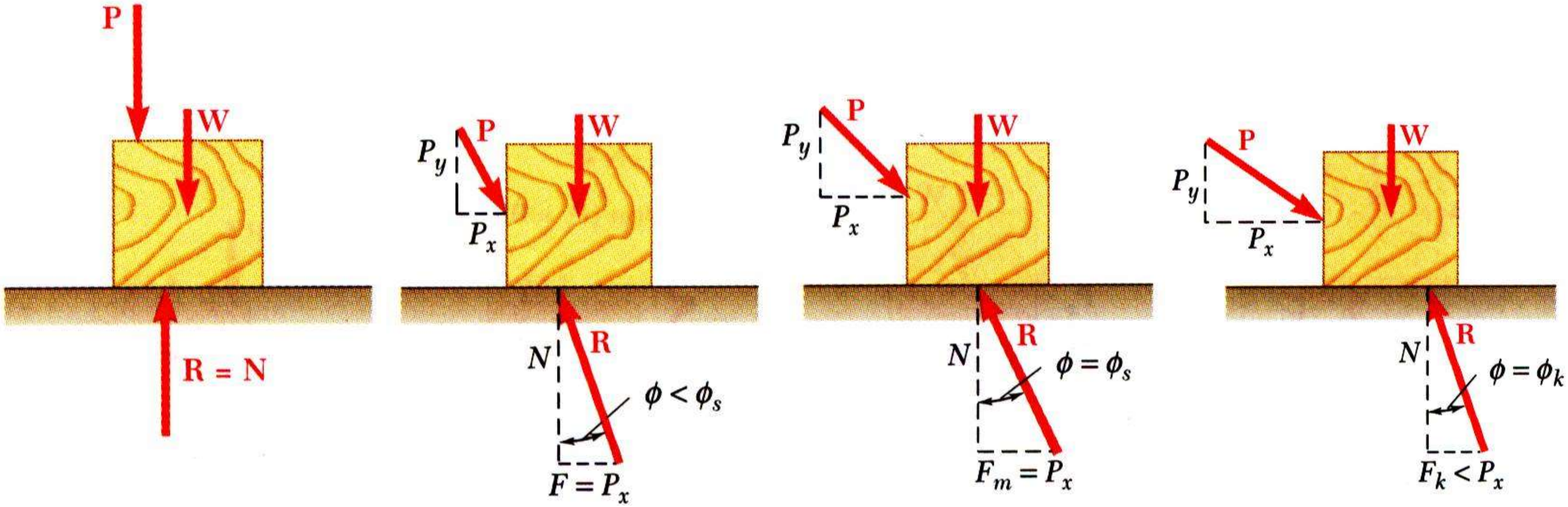
- **When P =200N**
- The motion impends. The body is in limiting equilibrium and hence the frictional force
- F = $\mu\ _{X}R_{N}$ =0.2x1000 = 200N.

- **When P = 300 N**
- Since the limiting friction is 200 N and the applied force is 300 N, the body moves towards right. Frictional force = p, $_{kX}R_{N}$ = 0.18x 1000 = 180N

- **Angle of friction**
- It is the angle between the normal reaction at the contact surface and the resultant of normal reaction and limiting friction. It is denoted by ϕ
- **tan ϕ = F /** $R_N = \mu \times R_N / R_N = \mu$



- **Angle of Friction**



- No friction
- No motion
- Motion impending
- Motion

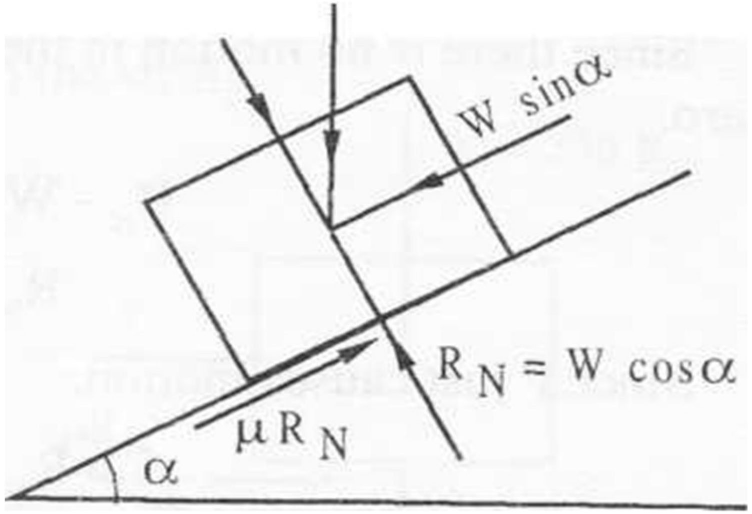
$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

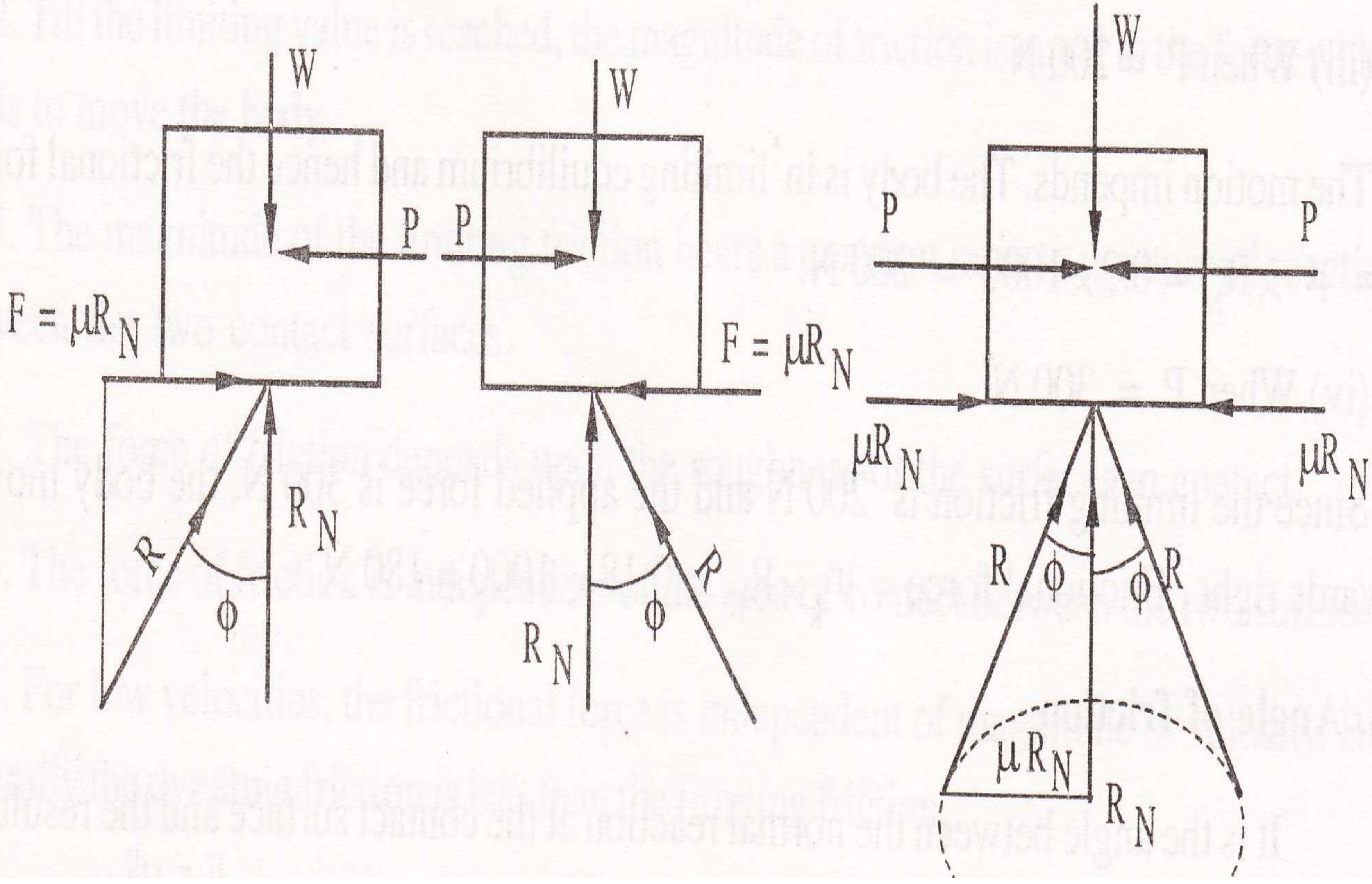
$$\tan\phi_k = \mu_k$$

- **Angle of repose**
- It is the maximum inclination of a plane on which a body can repose without applying external force.
- When motion impends,
- frictional force $F = \mu_X R_N = W \sin \propto$
- $R_N = W \cos\propto$
- $\mu\ W \cos\propto = W \sin \propto$
- $\tan s\propto = \tan\phi = \mu$
- $\propto = \phi$ **= angle of friction**



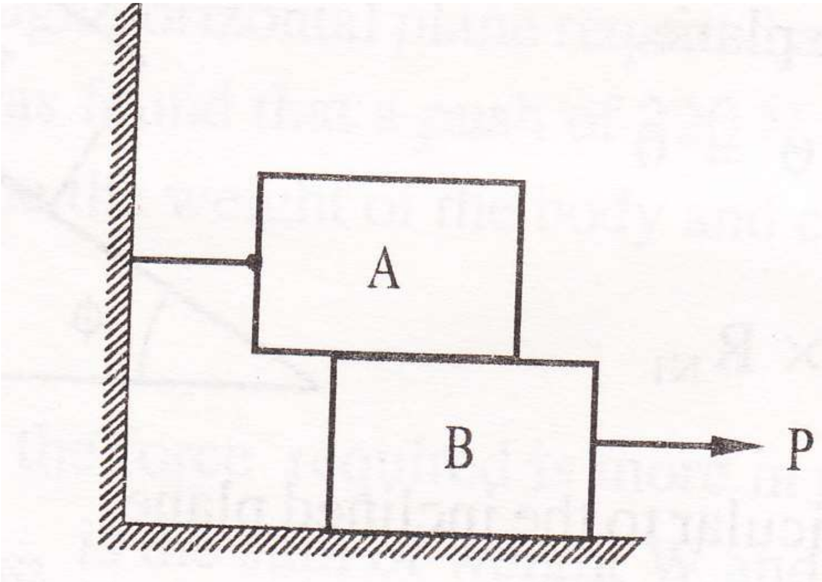
Cone of friction.

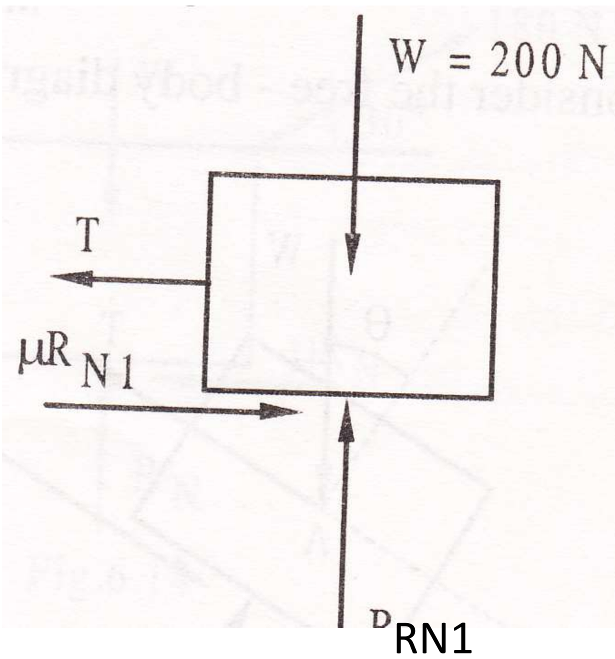
- Limiting equilibrium of a body kept on a horizontal surface.
- P be the applied force and R_N be the normal reaction.
- Frictional force $F = \mu_X R_N = W \sin \propto$
- Resultant of limiting friction and normal reaction makes an angle equal to angle of friction with the normal reaction.
- When the direction of external force is changed the direction of resultant changes but the angle between the normal reaction and the resultant force will be the same.
- Thus when the direction of external force is gradually changed through 360°, the resultant R generates a right circular cone with semi cone angle equal to ϕ.
- This cone is called friction cone or cone of friction.
- The axis of this cone will be the normal reaction and the generators are the resultant force and base radius is equal to the limiting frictional force.



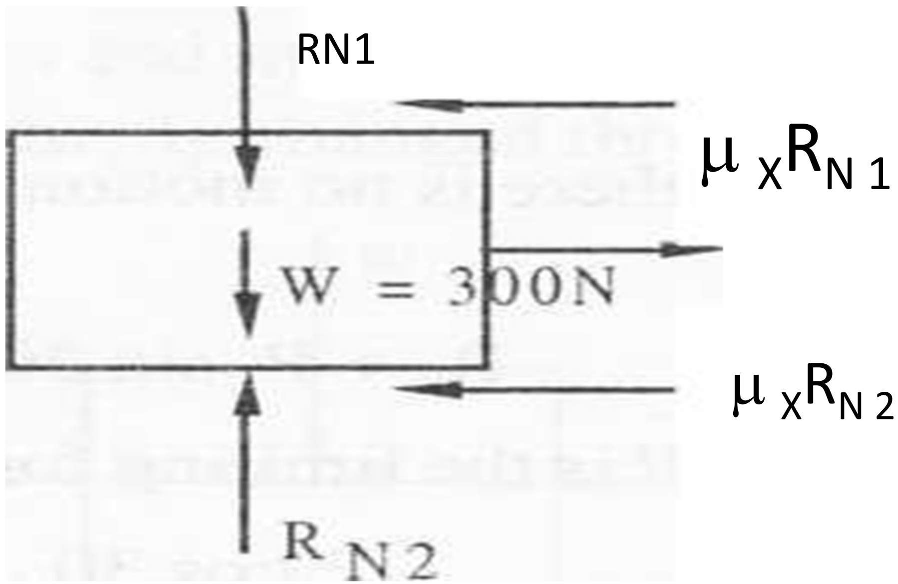
W
P
F = μR N
R
R N
ϕ
W
P
F = μR N
R N
R
ϕ
W
P
P
μR N
μR N
R
ϕ
ϕ
R
μ R N
R N

- Block A in fig weighs 200 N and block B weighs 300 N. Find the force P required to move block B. Assume the coefficient of friction for all surface as 0.3.





W = 200 N
T
μR N 1
RN1

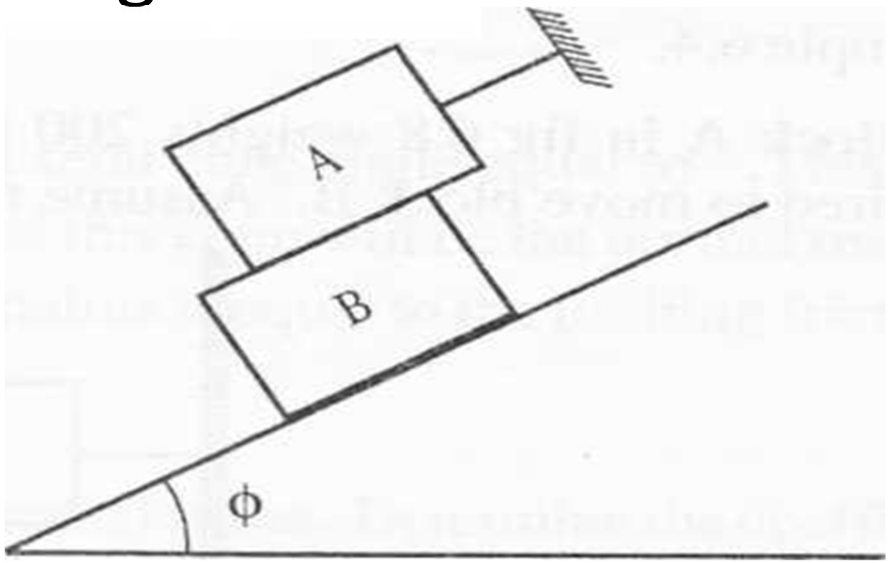


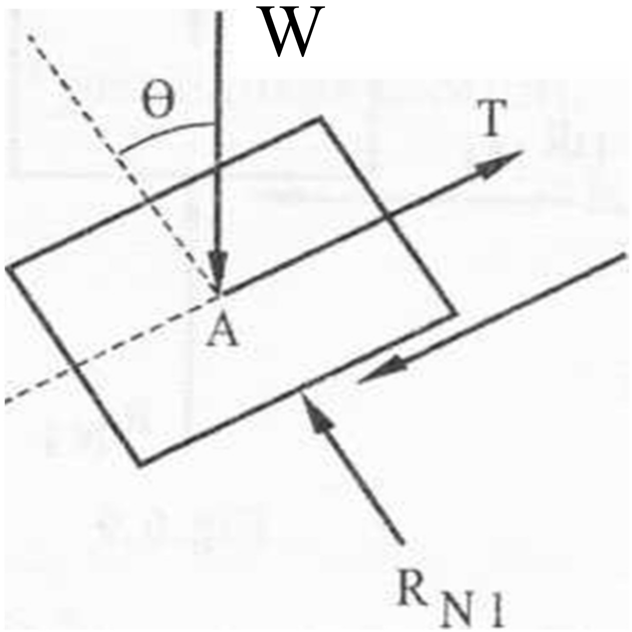
RN1
μ XRN 1
W = 300N
μ XRN 2
R N 2

- Upper Block A
- $R_{N1} – W = 0$
- $\mu_X R_{N1} - T = 0$
- T = 60 N ; Rn1 = 200 N

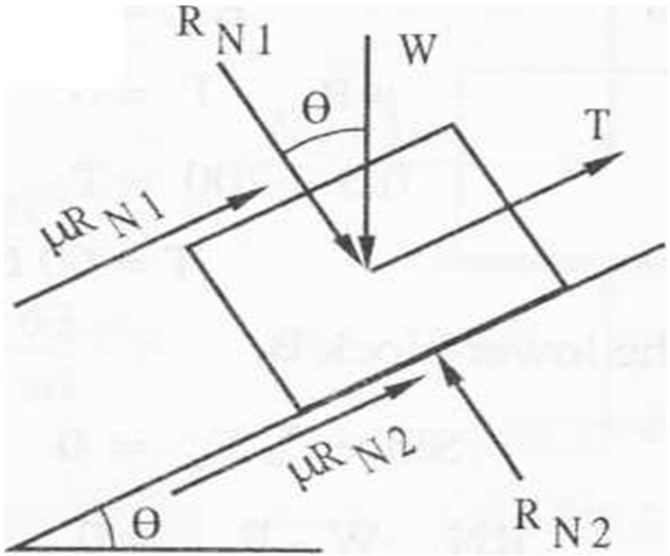
- Lower Block B
- Rn2 – W – Rn1 =0
- Rn2 = 500 N
- $P - \mu_X R_{N1} - \mu_X R_{N2} = 0$
- P = 210 N

- What should be the value of angle θ for the motion of the block B weighing 90 N to impend down the plane. The coefficient of friction for all surfaces of contact is 1/3 . Block A weights 30 N. The block A is held in position as shown in fig





W
Θ
T
A
R N1



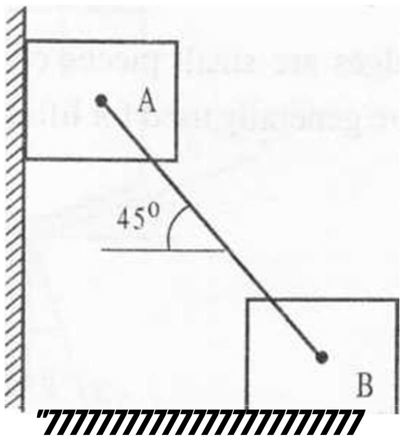
R N1
W
Θ
T
μR N1
μR N2
R N2
Θ

- Block A
  - Along the plane
  - $T - \mu_X R_{N1} - Wa \sin\theta = 0$
  - Perpendicular to plane
  - $R_{N1} - Wa \cos\theta = 0$

- Block B
  - Along the plane
  - $\mu_X R_{N2} - \mu_X R_{N1} - Wb \sin\theta = 0$
  - Perpendicular to plane
  - $R_{N2} - R_{N1} - Wb \cos\theta = 0$

- $\Theta = 29.03$

- Two identical blocks, A and B, of weight W are supported by a rigid bar inclined 45° with horizontal as shown in fig.
- If both the blocks are in limiting equilibrium, find the coefficient of friction, assuming it to be the same at the floor and the wall.



- Since the bar is in compression, the compressive force is directed towards the ends of the bar as shown in the free - body diagram of blocks A and B

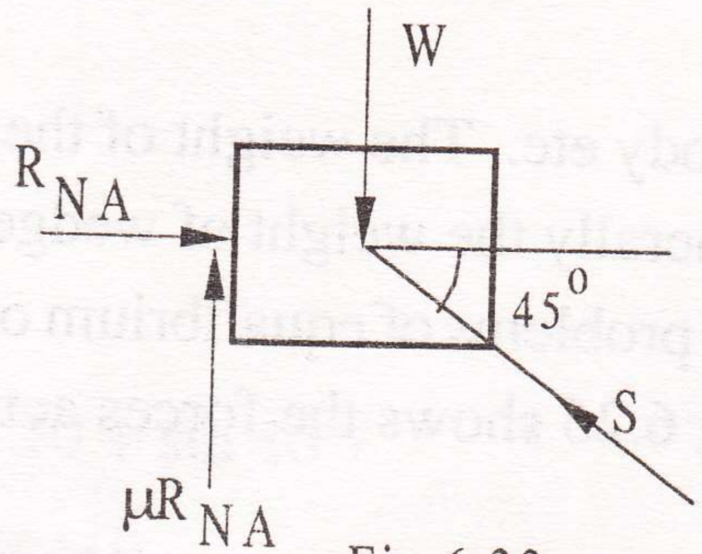
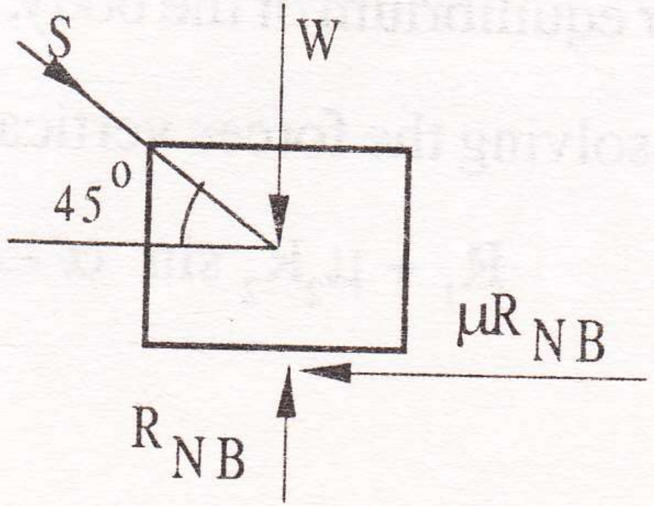
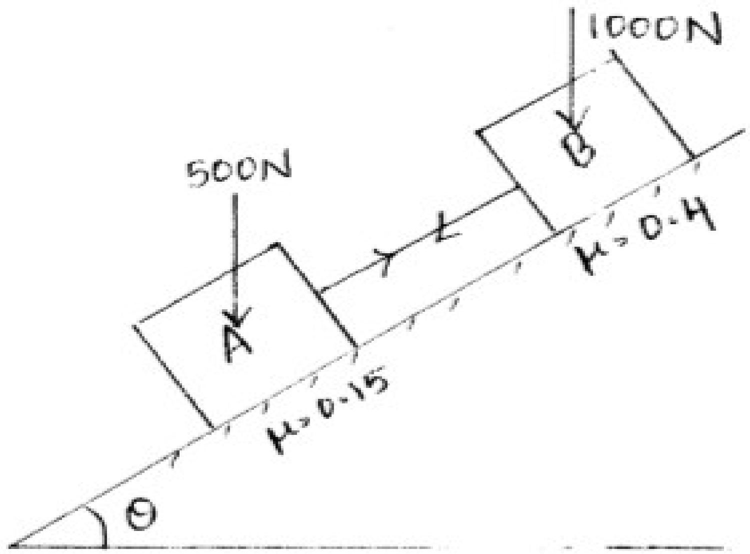


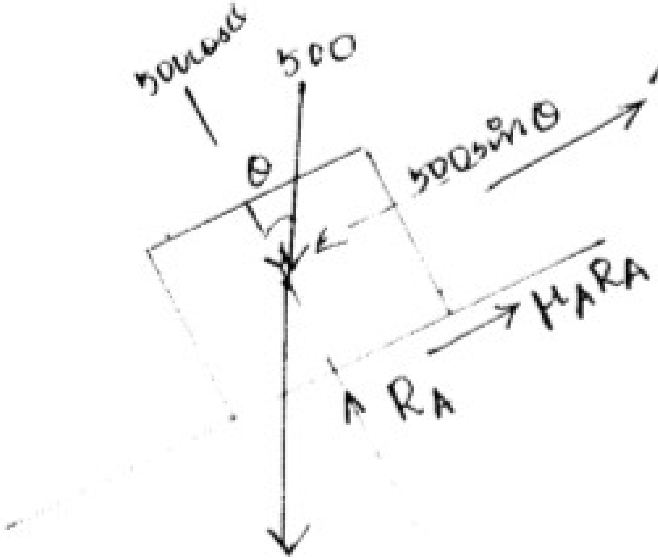
Fig.6.23

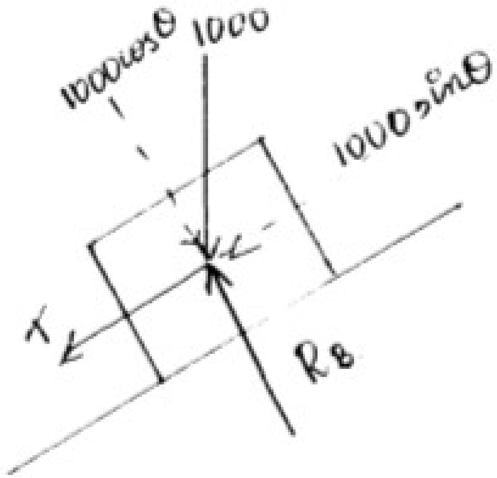


- μ = 0.414

- Two blocks A and B of weights 500N and 1000N are placed on an inclined plane. The blocks are connected by a string parallel to the inclined plane. If the coefficient of friction between the inclined plane and block A is 0.15 and that of block B is 0.4. Find the inclination of the plane when the motion is about to take place. Also calculate the tension in the string. Block A is below block B.


1000N
500N
A
B
T
μ = 0.15
μ = 0.4
θ


500cosθ
500
θ
500sinθ
T
μARA
RA


1000cosθ
1000
1000sinθ
T
RB

- Block A

- Along the plane:

$$500 \sin \theta = T + 0.15 \times 500 \cos \theta$$

- Perpendicular to plane:

$$500 \cos \theta = R_A$$

- Block B

- Along the plane:

$$T + 1000 \sin \theta = 0.4 \times 1000 \cos \theta$$

- Perpendicular to plane:

$$1000 \cos \theta = R_B$$

- Equating expressions for T:

$$500 \sin\theta - 75 \cos\theta = 400 \cos\theta - 1000 \sin\theta$$

$$1500 \sin\theta = 475 \cos\theta$$

$$\tan\theta = \frac{475}{1500}$$

$$\theta = 17.571$$

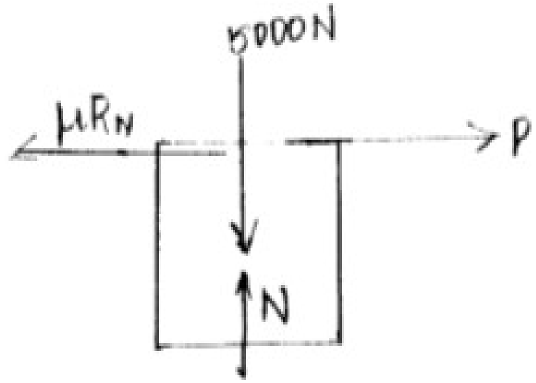
- $\sin\theta = 0.3$, $\cos\theta = 0.95$
- $T = 500 \times 0.3 - 75 \times 0.95 = 78.5$ N

- A uniform wooden cube of side 1 m and weighing 5000N rest on its side on a horizontal plane. Find what maximum horizontal force can be applied at the top edge of cube to make it slide without overturning.

$P = \mu R_N$

$= 0.25 x 5000$

$= 1250N$



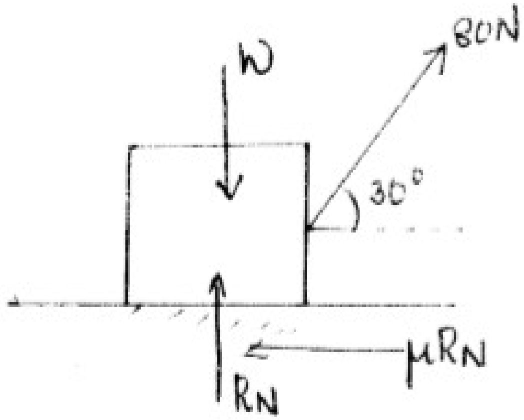
- A body of weight W is resting over a surface having coefficient of friction μ . It is subjected to a pull of 80N inclined at an angle of 30^0 with the horizontal so as to just move it. Similarly a push of 100N inclined at 30^0 with the horizontal will the cause the body to move. Find μ

- $R_N + 80\sin30^0 = W$

  $R_N = W - 40$

- $\mu R_N = 80\cos30^0$

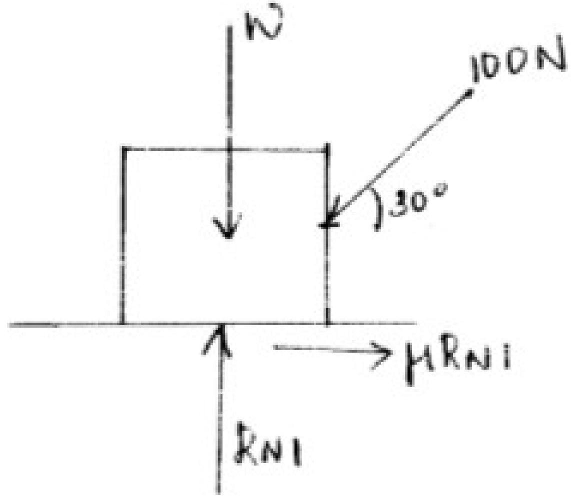
  $R_N = \dfrac{80\cos300}{\mu}$



$W = 40 + \dfrac{80\cos300}{\mu}$

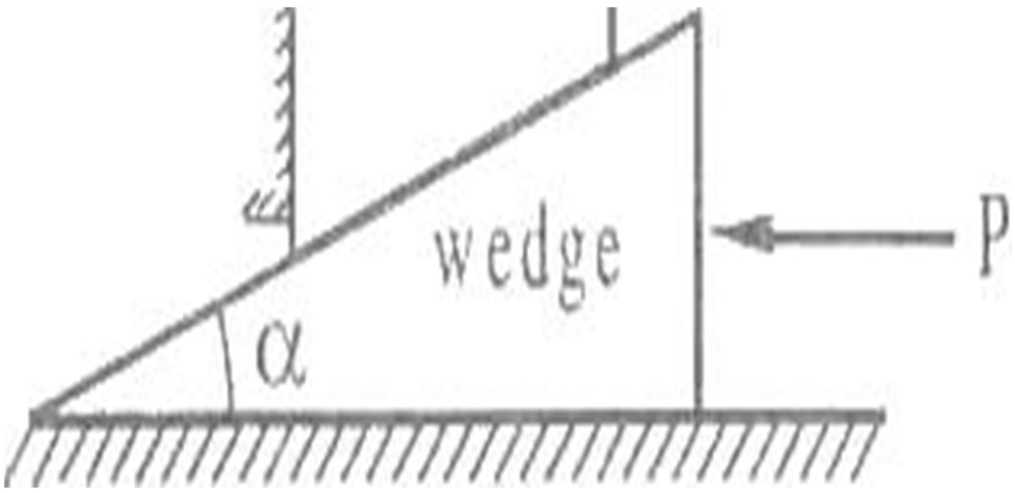
$W = 400N$

- $W + 50 = R_{N1}$
- $100\cos 30^o = \mu R_{N1}$
- $R_{N1} = \dfrac{100\cos 30^o}{\mu}$
- $W = R_{N1} - 50$
- $40 + \dfrac{80\cos 300}{\mu} = \dfrac{100\cos 30^o}{\mu} - 5($
- $90 = \dfrac{20\cos 30^o}{\mu}$
- $\mu = 0.192$

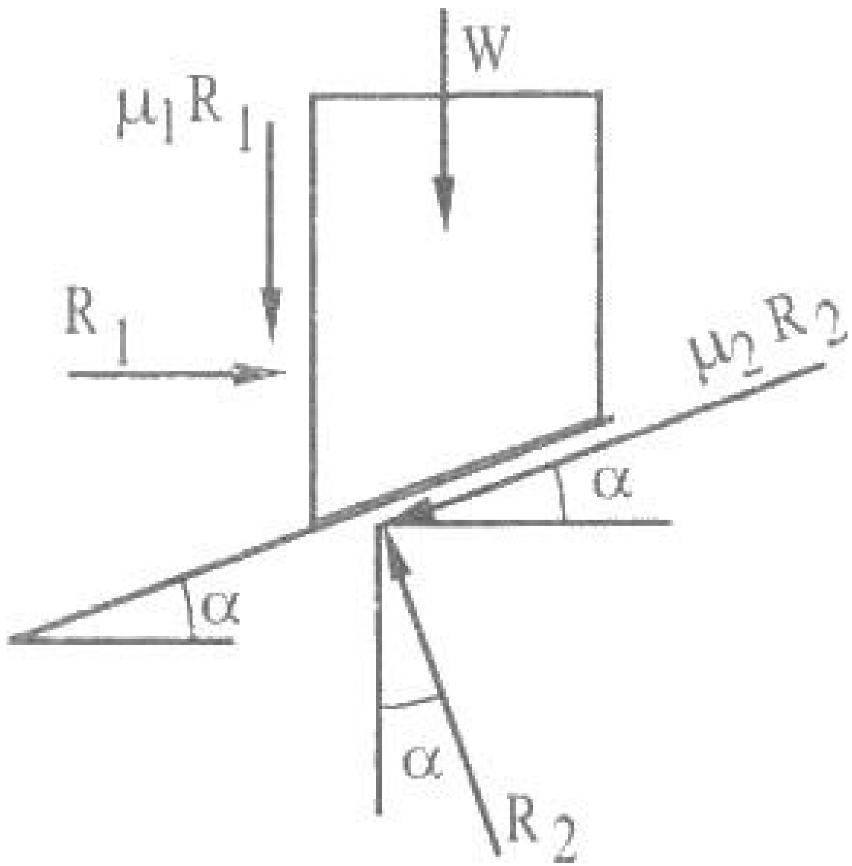


Wedge friction.

- Wedges are small pieces of materials with triangular or trapezoidal cross section.
- They are generally used for lifting heavy weights, for slight adjustments in the position of a body etc.
- The weight of the wedge is very small compared to the weight lifted.
- Hence generally the weight of wedge will be neglected.

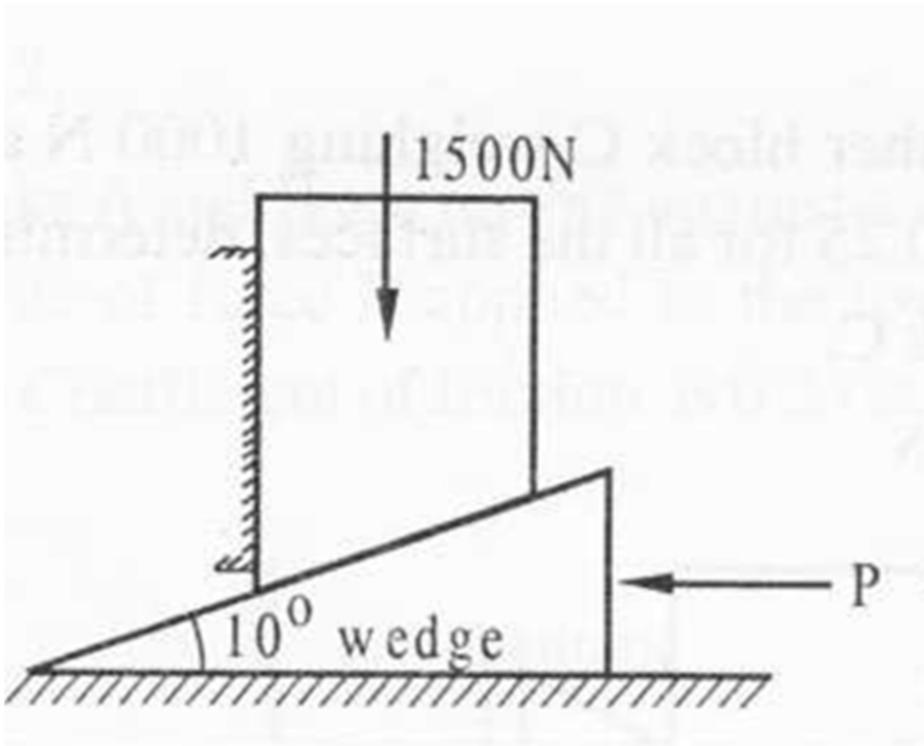


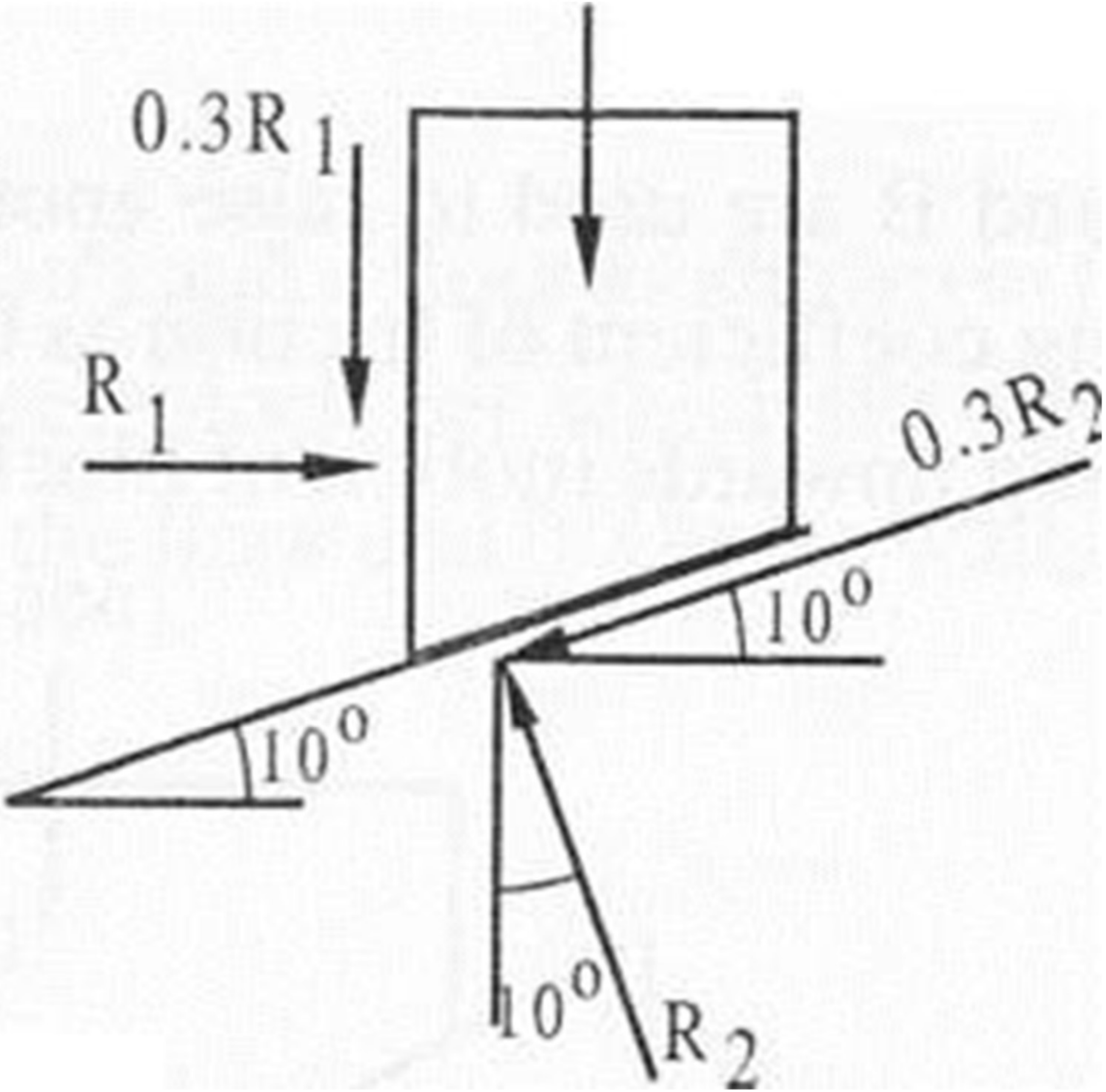
wedge
P
α



W
μ1 R1
R1
μ2 R2
α
α
α
R2

- Find the horizontal force P on the 10° wedge shown in fig. to raise the 1500 N load. The coefficient of friction is 0.3 at all contact surfaces

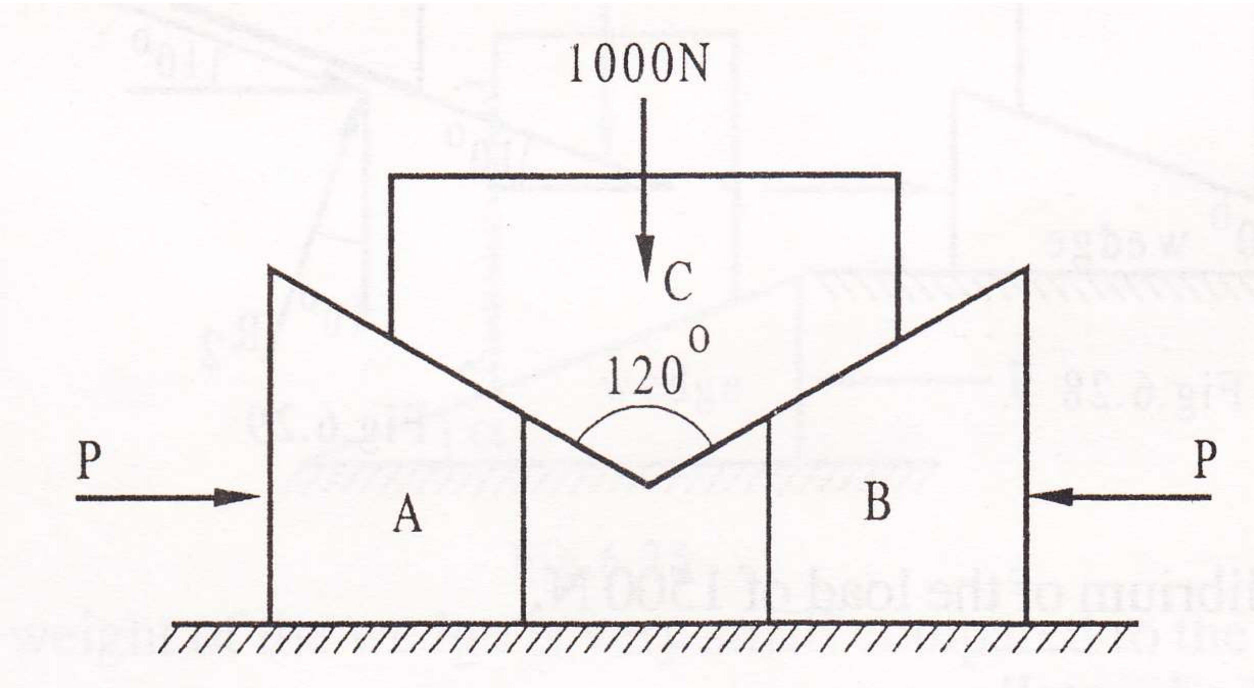


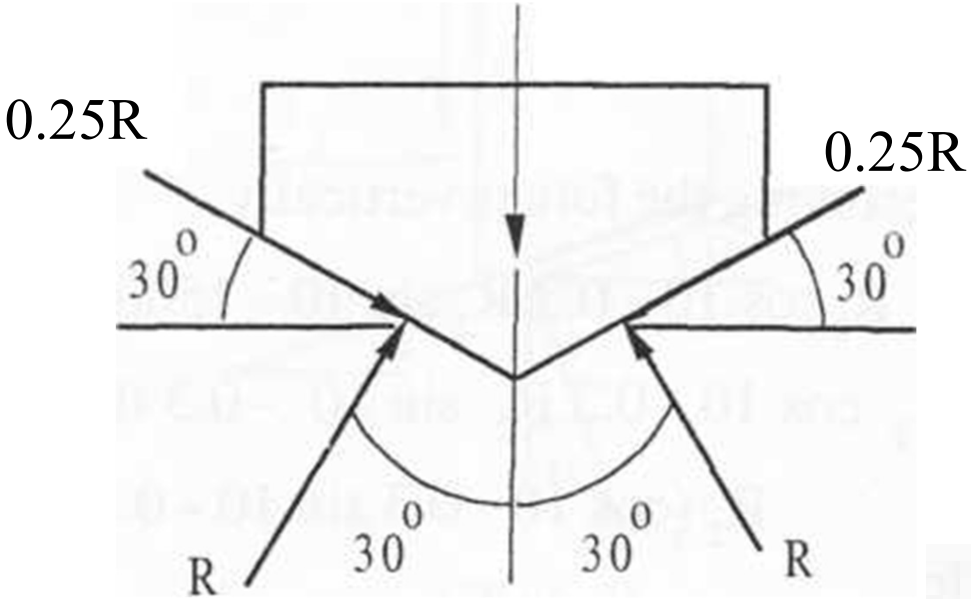

0.3R1
R1
0.3R2
10°
10°
10°
R2

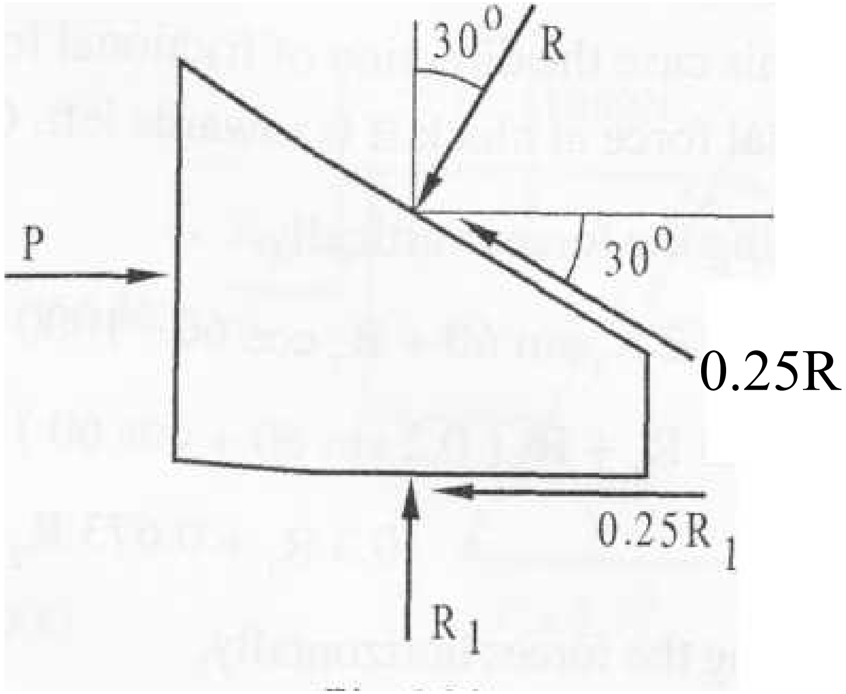
- Equilibrium of the load of 1500 N.
- Resolving the forces horizontally
  - $R1 - 0.3\ R_2 \cos 10 - R_2 \sin 10 = 0$
  - $R1 = 0.47\ R2$
- Resolving the forces vertically
  - $R_2 \cos 10 - 0.3\ R_2 \sin 10 - 1500 - 0.3\ R1 = 0$
  - $R2 = 1894.63$ N

- equilibrium of wedge.
- resolving the forces vertically,
  - $R_3 + 0.3\ R_2 \sin 10 - R_2 \cos 10 = 0$
  - $R3 = 1767.15$ N
- Resolving the forces horizontally
  - $0.3\ R_2 \cos 10 + R_2 \sin 10 + 0.3\ R_3 - P = 0$
  - $P = 1418.90$ N

- Two wedges A and B are used to raise another block C weighing 1000 N as shown in fig. Assuming coefficient of friction as 0.25 for all the surfaces, determine the value of P for impending upwards motion of block C.

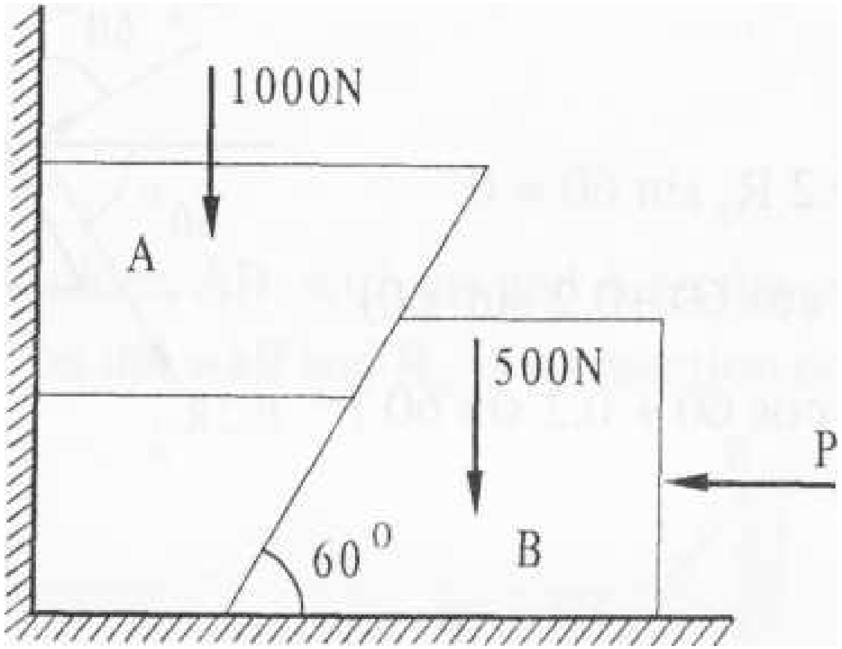



0.25R
0.25R
30°
30°
30°
30°
R
R

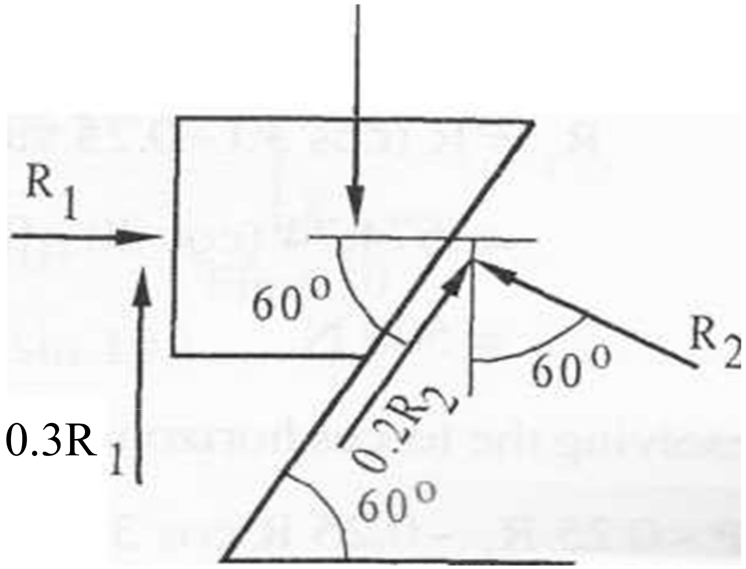

30°
R
P
30°
0.25R
$0.25R_1$
R_1

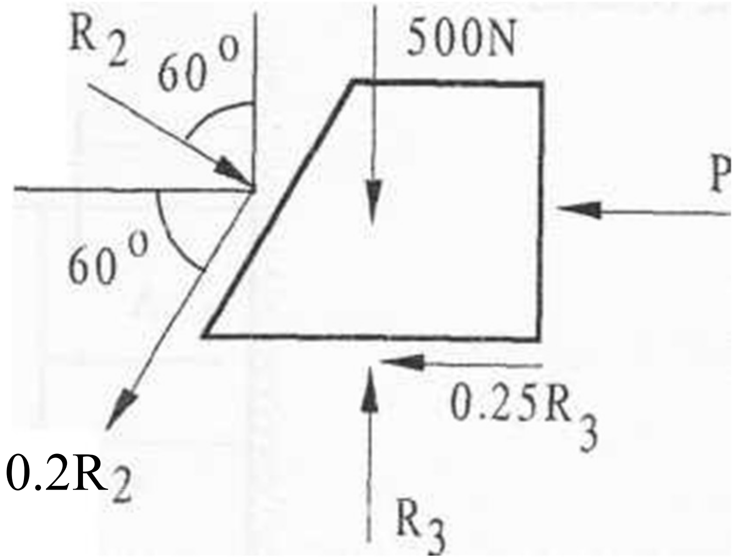
- Because of symmetry, the reaction at the contact surface between A and C will be 1000 N same as that between B and C.
- Let this reaction be R
- Equilibrium of block C.
  - Resolving the forces vertically,
  - R cos 30 + R cos 30 - 1000 - 2x0.25 R sin 30 = 0
  - R = 674.74 N
- equilibrium of wedge A.
  - Resolving the forces vertically
  - R_1 + 0.25 R sin 30 - R cos 30 = 0 R, = R (cos 30 - 0.25 sin 30)
  - R1 =500 N.
  - Resolving the forces horizontally,
  - P - 0.25 R1 - 0.25 R cos 30 - R sin 30 = 0
  - P =608.46 N

- Two blocks A and B are resting against a wall and a floor as shown in fig
- Find the range of value of force P applied to the lower block for which the system remains in equilibrium. Coefficient of friction is 0.25 at the floor and 0.3 at the wall and 0.2 between the blocks



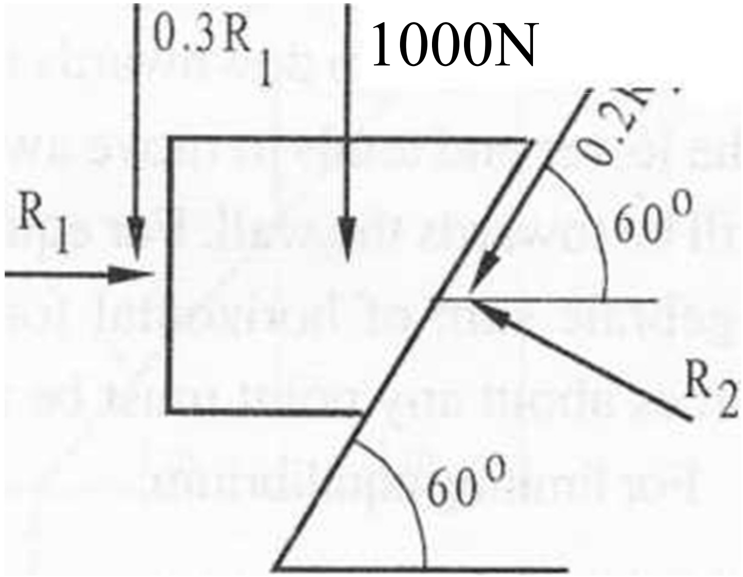
- If the applied force is less than the minimum force required to keep the system in equilibrium, then the block A will move downwards and block B will move towards right.
- If the applied force is more than a certain value, then the block A will move upwards and block B will move towards left.
- The minimum value of P is the force required to prevent the block A from moving down and the maximum value of P is the force required just to move the block upwards.

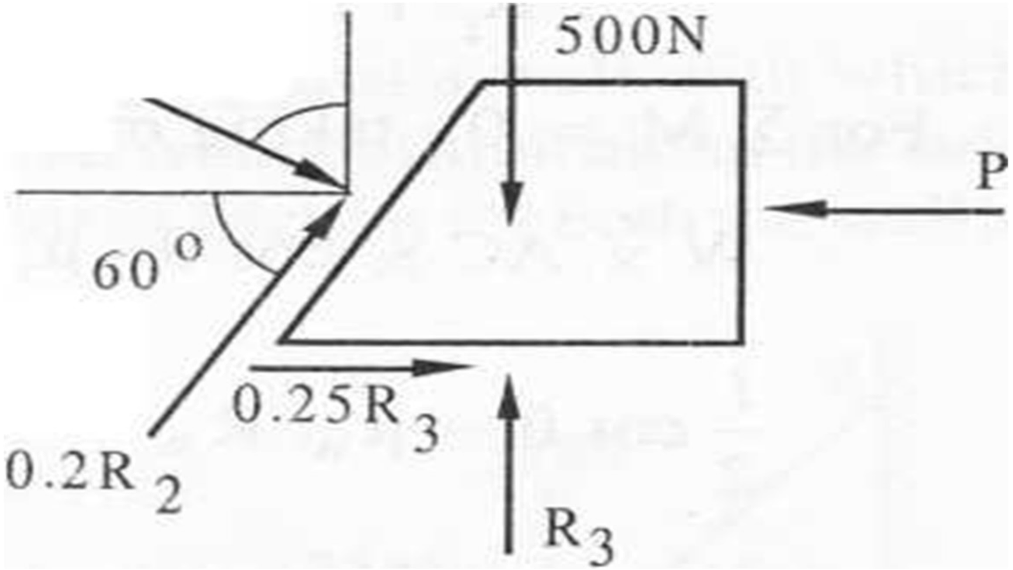

R1
60°
R2
60°
0.2R2
0.3R1
60°


R2
60°
500N
P
60°
0.25R3
0.2R2
R3

- Minimum value of force P
- In this case the direction of frictional force at block A is upwards and the direction of frictional force at block B is towards left.
- Consider the equilibrium block A.
- Resolving the forces vertically,
- $0.3\ R1 + 0.2\ R_2 \sin 60 + R_2 \cos 60 - 1000 = 0$
- Resolving the forces horizontally,
- $R1 + 0.2\ R_2 \cos 60 - R_2 \sin 60 = 0$
- $R1 = 0.766\ R_2$.
- Substituting this value of R1 in eqn (i)
- $R_2 = 1107.66$ N
- $673\ R_2 = 1000$ —(i)

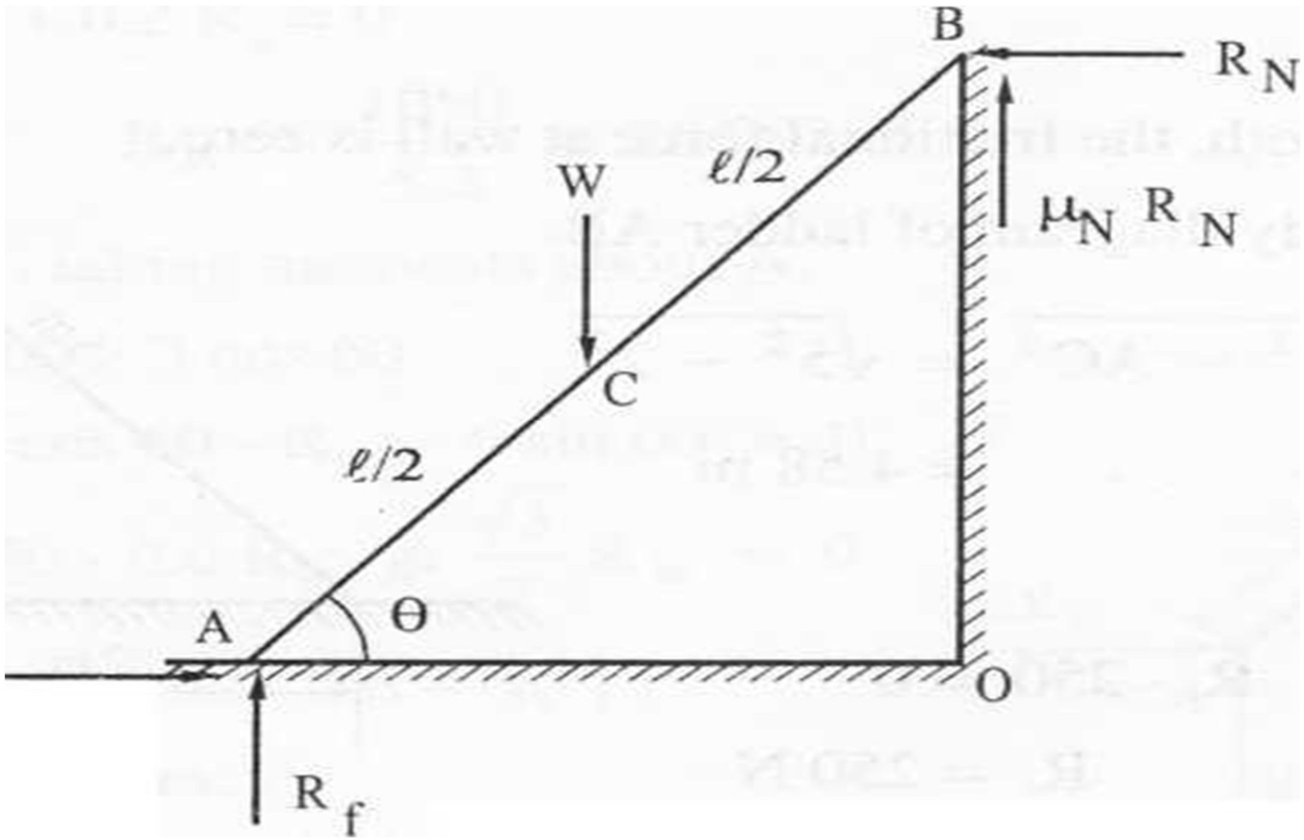
- Consider the equilibrium of block B
- Resolving the forces vertically,
- R_3 - 500 - R_2 cos 60 - 0.2 R_2 sin 60 = 0
- R_3 = 1245.68 N
- Resolving the forces horizontally,
- R_2 sin 60 - 0.2 R_2cos 60 - 0.25 R_3 – P= 0
- P = 537.05 N
- Force P required to just move the block A upwards


0.3R1
1000N
0.2R
R1
60°
R2
60°


500N
P
60°
0.25R3
0.2R2
R3

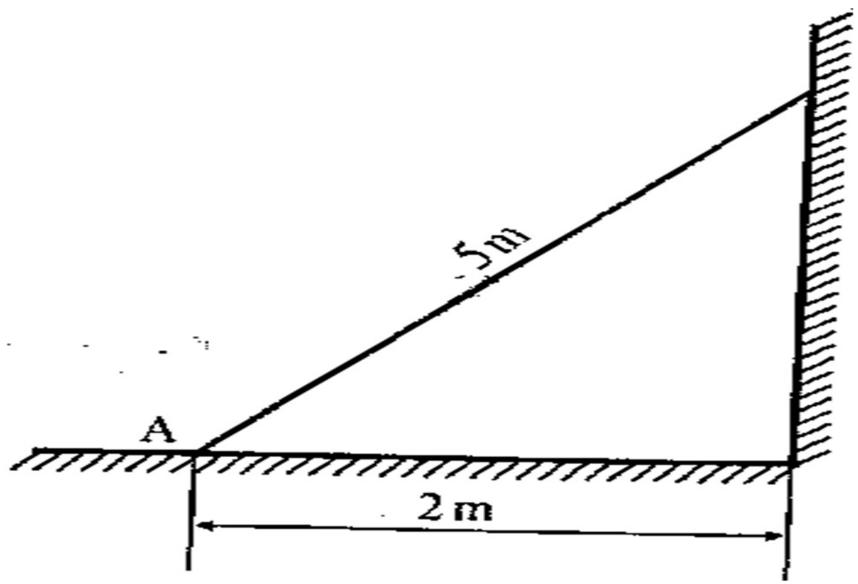
- Consider the equilibrium of block A
- Resolving the forces horizontally,
- $R1 - 0.2 R_2 \cos 60 - R_2 \sin 60 = 0$
- Resolving the forces vertically,
- $R_2 \cos 60 - 0.2 R_2 \sin 60 - 0.3 R, - 1000 = 0$
- Solve
- R2 = 27027 N
- Consider the equilibrium of block B.
- Resolving the forces vertically,
- $R_3 + 0.2 R_2 \sin 60 - R_2 \cos 60 - 500 = 0$
- R3 = 9337.8 N

- **Ladder friction.**
-



- **Ladder friction.**
- Ladder, AB, with the end A on the ground and end B to the wall.
- The ladder exerts a force on the wall and R_w is its reaction on the ladder.
- Similarly the ladder exerts a force on the ground and R_f is its reaction on the ladder.
- The upper end B of the ladder tends to slip downwards and hence the force of friction will be vertically upwards.
- The lower end tends to move away from the wall and hence the direction of friction force will be towards the wall.
- For equilibrium of ladder the algebraic sum of vertical forces and algebraic sum of horizontal forces must be zero.
- Also the sum of moments of all the forces about any point must be zero For limiting equilibrium

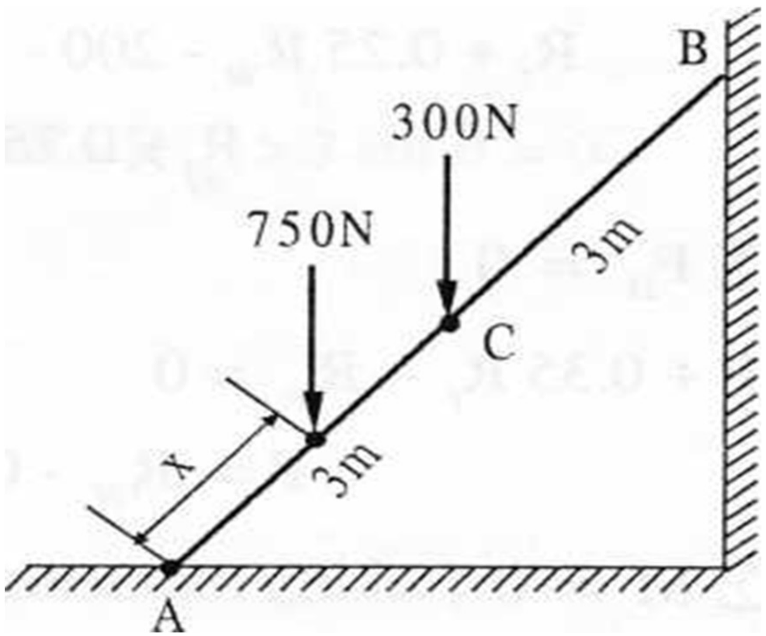
- A uniform ladder 5 m long, weighing 250 N, is placed against a smooth vertical wall with its lower end 2 m from the wall. The coefficient of friction between the ladder and floor is 0.25. Show that the ladder will remain in equilibrium in this position



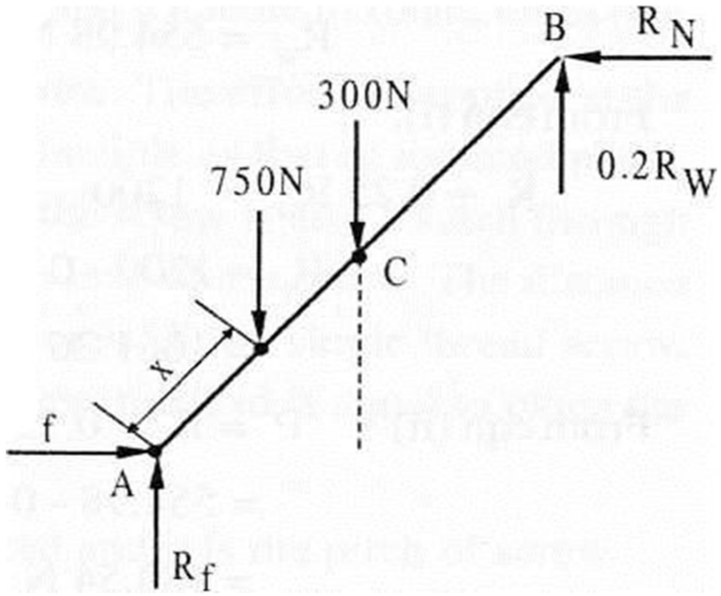
- Since the wall is smooth, the frictional force at wall is zero.
- free-body diagram of ladder AB.
- BC = 4.58 m
- $\sum Fv = 0$
- $Rf - 250 = 0$
- $R_f = 250N$

- The limiting frictional force is $\mu R_N = 0.25 _x 250 = 62.5$ N
- Taking moments about B,
- $R_{f\ .}$ AC- 250. 1 –F. BC = 0
- $250x2 - 250x1 = F_X 4.8$
- F = 52.08 N
- Since the frictional force at A is less than the limiting frictional force of 62.5 N, the ladder will remain in equilibrium.

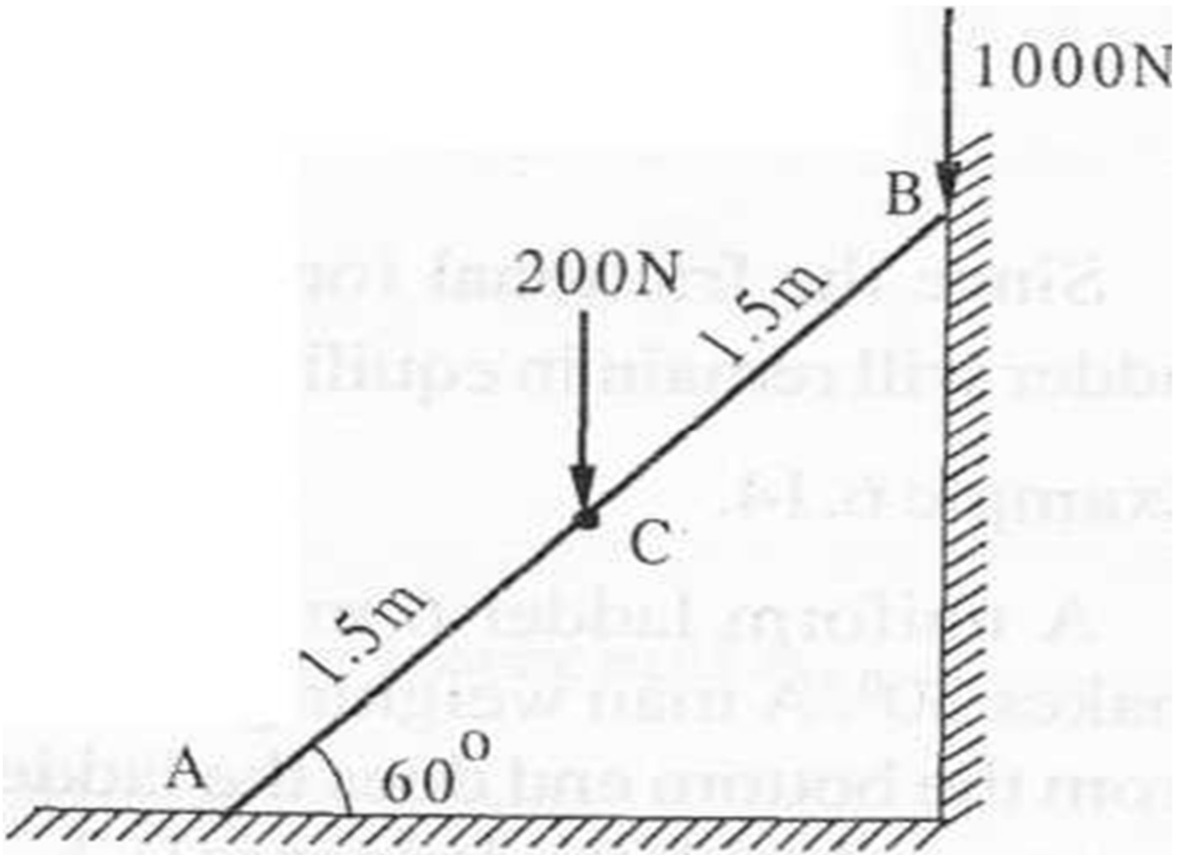
- A uniform ladder 6 m long weighing 300 N, is resting against a wall with which it makes 30°. A man weighing 750 N climbs up the ladder. At what position along the ladder from the bottom end does the ladder slips? The coefficient of friction for both the wall and the ground with the ladder is 0.2



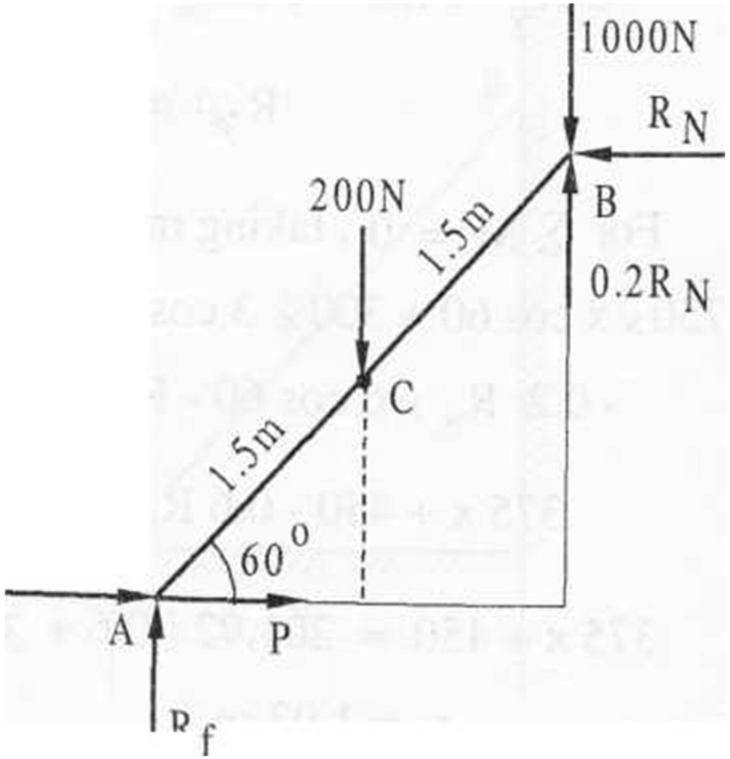
- Consider the limiting equilibrium of the ladder
- $\sum Fh = 0$
- $0.2\ Rf - Rw = 0$
- $\sum Fv = 0$
- $Rf - 750 - 300 + 0.2\ Rw = 0$
- $Rw = 201.9$ N
- $\sum M = 0$ about A
- $750 \cdot X \cdot \cos 60 + 300 \cdot 3 \cos 60 - 0.2\ Rw\ 6 \cos 60 - Rw \cdot 6 \sin 60 = 0$
- Distance $x = 1.92$ m

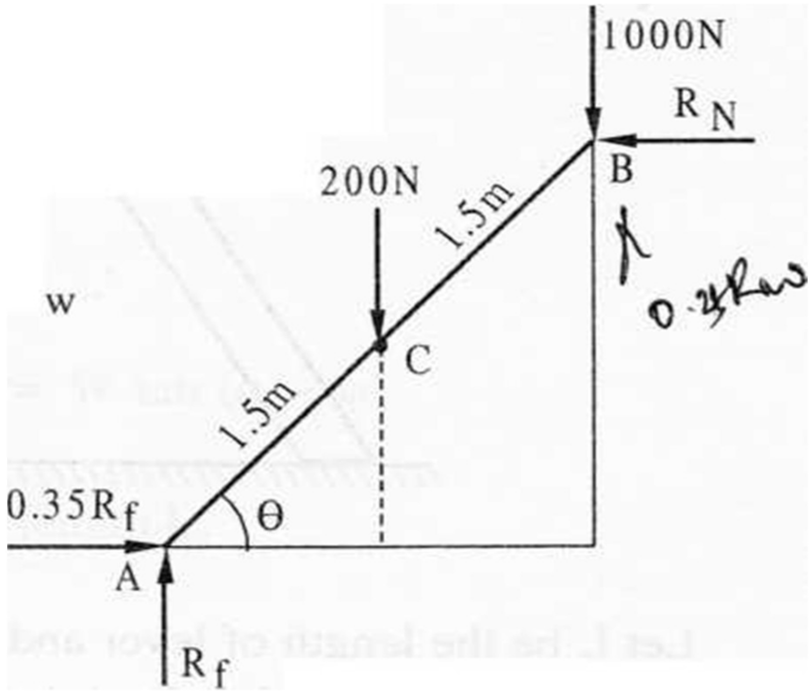


- A uniform ladder 3 m long weighs 200 N. It is placed against a vertical wall with which it makes an angle of 30°. The coefficient of friction between the wall and the ladder is 0.25 and that between the floor and ladder is 0.35. The ladder in addition to its own weight has to support a load of 1000 N at its top end.
- Find, (i) the horizontal force P to be applied to the ladder at the floor level to prevent slipping.
- (ii) If the force P is not applied, what should be the minimum inclination of the ladder with the horizontal so that there is no slipping of it with the load at its top end.


1000N
B
200N
1.5m
C
1.5m
A
60°

- Inclination of ladder with horizontal = 60
- Case-1
- Consider the limiting equilibrium of the ladder
- For $\Sigma F_v = 0$,
  - $R_f + 0.25\ Rw - 200 - 100 = 0$
- For $\Sigma F_h = 0$,
  - $P + 0.35\ R_f - R_w = 0$
- For $\Sigma M = 0$, taking moments about A,
  - $200x\ 1.5 \cos 60 + 1000x\ 3 \cos 60 - 0.25\ R_w\ 3 \cos 60 - R_{w\ x}\ 3 \sin 60 = 0$
  - Rw = 554.98 N
- From 1
- Rf = 1061.26 N
- From 2
- P = 183.54 N


1000N
R N
200N
1.5m
B
0.2R N
C
1.5m
60°
A
P
R f


1000N
R N
200N
1.5m
B
C
1.5m
0.35R f
Θ
A
R f

- Case (ii) Let the required inclination of ladder with horizontal be θ
- For Σ F_v =0,
- R_f - 200 - 1000 + 0.25 R_w = 0
- For Σ F_h=0,
- 0.35 R_f - Rw =0
- Rf= 2.86 Rw
- Rw = 385.5 using 1
- For Σ M = 0 , taking moments about A
- 200 x 1.5 cosθ+ 1000 x 3 cos θ - 0.25 Rw x 3 cos θ - Rw x3 sin θ = 0
- Θ = 70.22