MODULE IV

STRESSES IN BEAMS OF SYMMETRICAL CROSS SECTIONS

BENDING STRESSES IN BEAMS

Introduction

- Due to external load, SF&BM at all sections
- Beams undergo deformation- to resist these

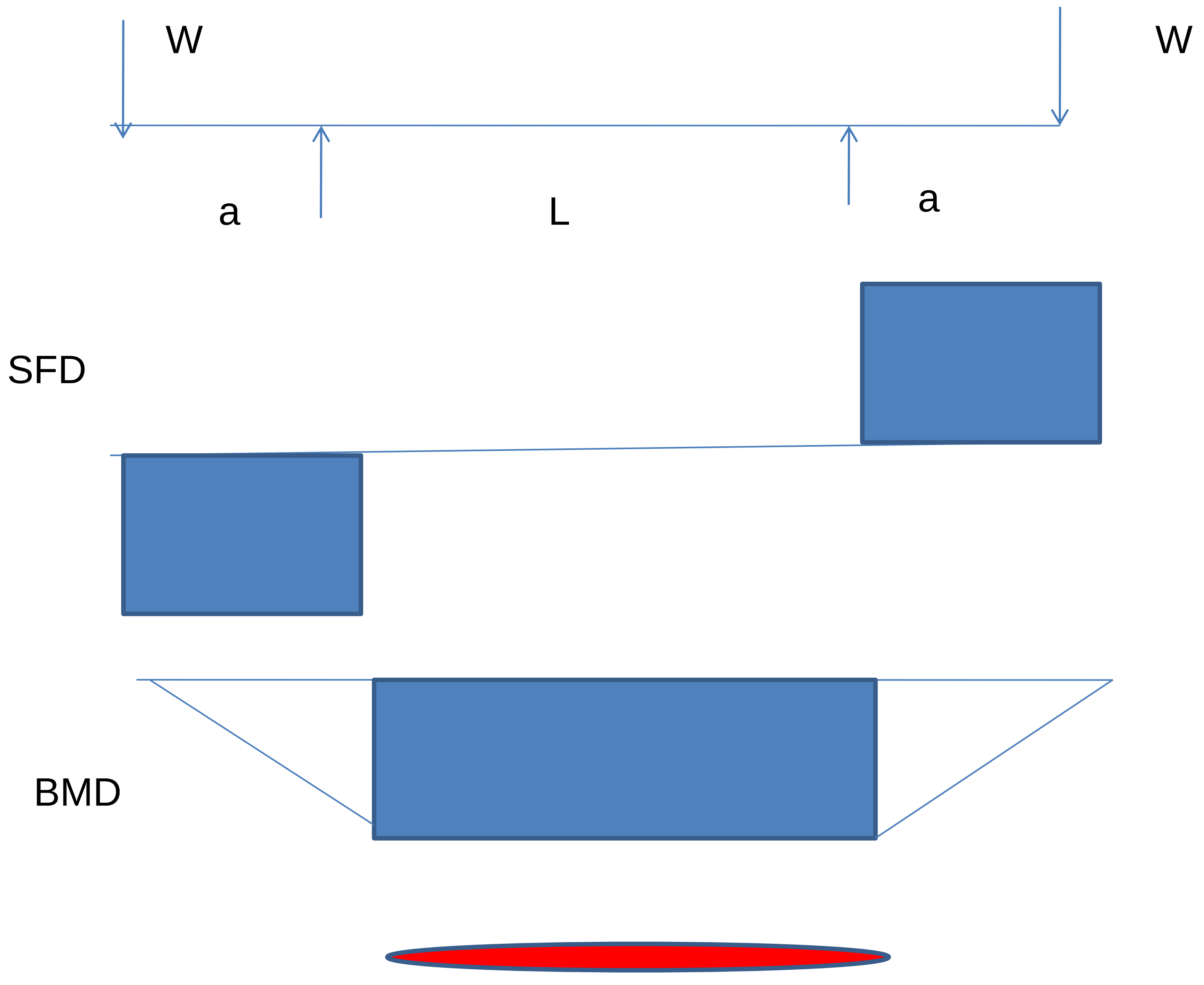
Stresses will be developed

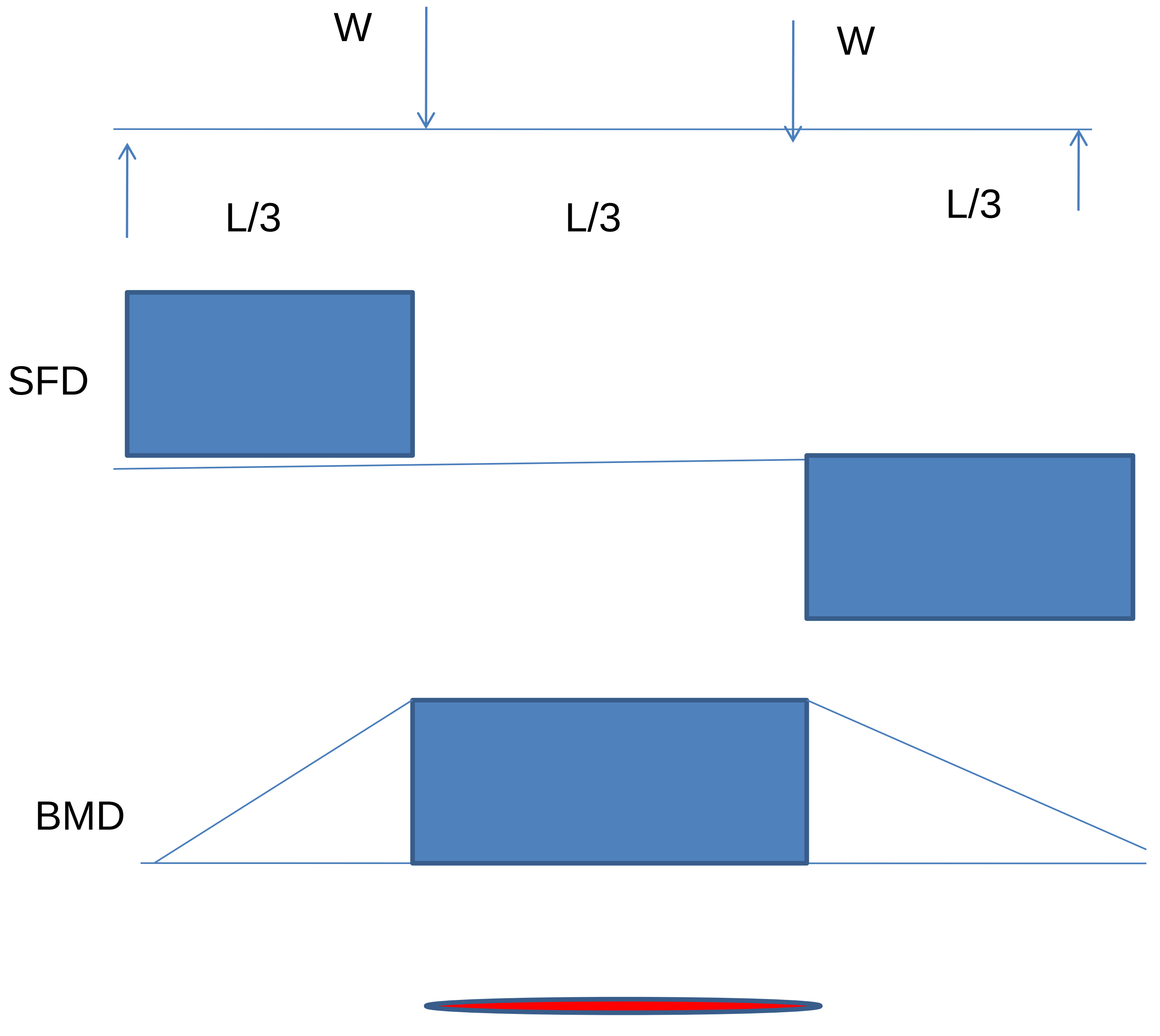
✓ Bending Stresses & Shear stresses

Pure bending (simple bending)

- If a length of a beam is subjected to constant BM

and zero SF; STRESSES will be developed in that length due to B.M. only, that region in simple bending


W
W
a
L
a
SFD
BMD



W
W
L/3
L/3
L/3
SFD
BMD

THEORY OF SIMPLE BENDING -ASSUMPTIONS

1. Material of the beam is homogeneous and isotropic.
2. Modulus of elasticity is the same in tension and compression.
3. Transverse sections which were plane before bending, remain plane after bending
4. Beam is initially straight and all longitudinal filaments bend into circular area with a common centre of curvature.

ASSUMPTIONS(contd…)

5. Radius of curvature is large compared with the dimensions of the cross section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.
7. Stress induced is proportional to strain and stresses developed are within elastic limit

Theory of Simple Bending

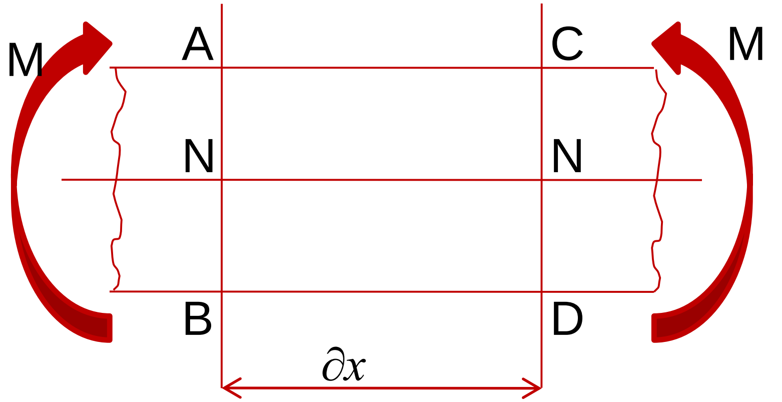


fig.1 **Before bending**

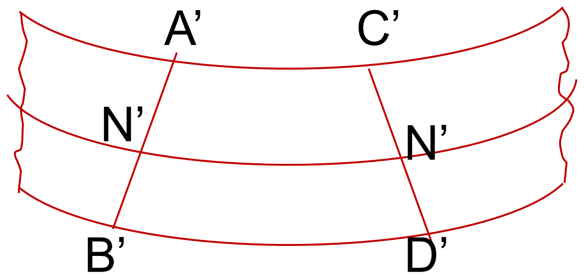


fig.2 **After bending**

The fig.1 shows a part of beam subjected to simple bending. Consider 2 sections AB and CD which are normal to the axis of the beam N-N. Due to the action of the BM, the part of length ∂x will be deformed.

Top layer AC has deformed to A'C' and is shortened.

Bottom layer BC has deformed to B'C' and is elongated.

In fig.2 it is clear that some of the layers have been shortened while some of them are elongated.

At a level between top and bottom of beam, there will be a layer which is neither shortened nor elongated. This layer is known as ***neutral layer or neutral surface***. This layer is shown by **N'-N'** and **N-N**.

Line of intersection of the neutral layer on a cross section of a beam is known as ***neutral axis.***

Layers above N-N have been shortened and those below, have been elongated.

Due to the decrease in lengths of the layers above N-N, these layers will be subjected to **compressive stresses**.

Due to the increase in the length of layers below N-N, these layers will be subjected to **tensile stresses.**

Top layer is shortened maximum. As we proceed to layer N-N, the decrease in the length of layer decreases.

At the layer N-N, there is no change in length.

Ie., compressive stress will be maximum at the top layer.

Similarly increase of length will be maximum at the bottom layer.

As we proceed from bottom layer to layer N-N, increase in length of layers decreases.

Amount by which a layer increases or decreases in length, depends upon the position of the layer with respect to N-N.

This theory of bending is called ***theory of simple bending***.

Expression for Bending Stress

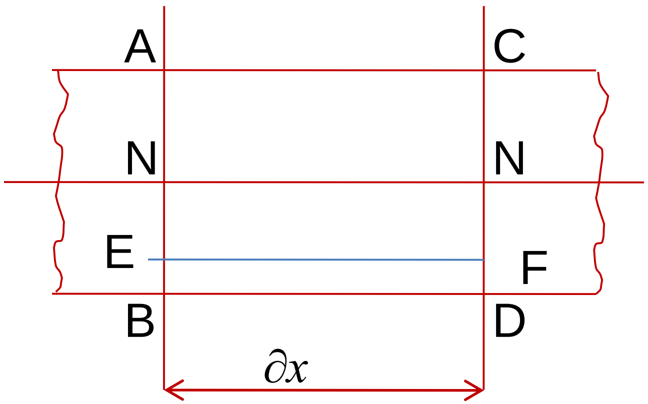


Fig3 a

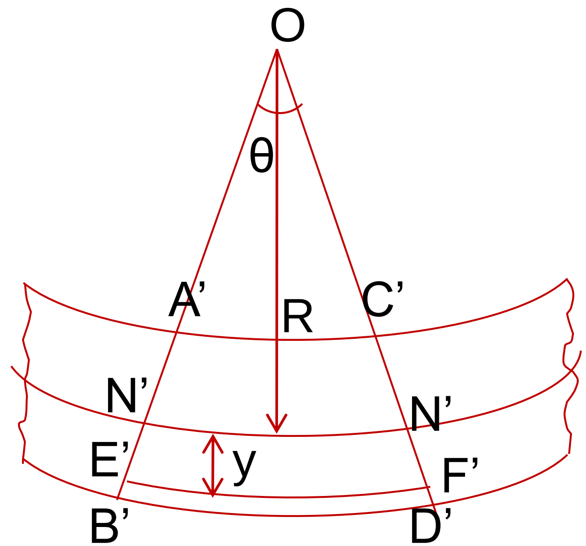


Fig 3b

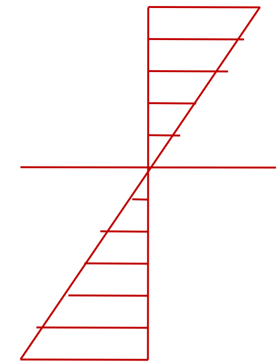


Fig 3cStress Diagram

Fig. 3a shows a small length ∂x of a beam subjected to a simple bending. Due to the action of bending, the part of length ∂x will be deformed as shown in the fig.3b.

Let A'B' and C'D' meet at O.

Let R = Radius of neutral layer N'N'

θ = Angle subtended at O by A'B' and C'D'

Strain variation along the Depth of Beam

Consider a layer EF at a distance y below the neutral layer NN. After bending this layer will be elongated to E'F'.

Original length of layer, EF = ∂x

Also length of neutral layer, NN = ∂x

After bending, length of neutral layer N'N' will remain unchanged. But length of layer E'F' will increase.

$$N'N' = NN = \partial x$$

From fig 3b, $N'N' = R\theta$ And $E'F' = (R+y)\theta$

But $N'N' = NN = \partial x$

Hence, $\partial x = R\theta$

Increase in the length of the layer EF = $E'F' – EF = (R+y)\theta – R\theta = y\theta$

Strain in layer EF = (Increase in length)/(Original length)

$$= y\theta/EF = y\theta/R\theta = y/R$$

As R is constant, *strain in a layer is proportional to its distance from neutral axis*.

variation of strain along the depth of the beam is linear.

Stress Variation

Let, σ = Stress in the layer EF

E = Young's modulus of the beam

Then, E = (Stress in the layer EF)/(Strain in the layer EF)

$= \sigma/(y/R)$

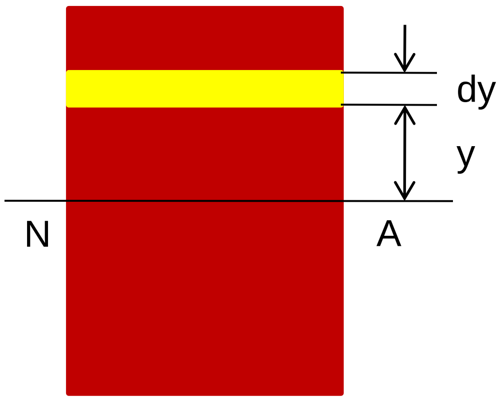
$\sigma = E(y/R) = (E/R)y$

$\sigma/y = E/R$

Since E and R are constant,

stress in any layer is directly proportional to the distance of the layer from the neutral layer.

Neutral axis- line of intersection of neutral layer with transverse section (NA)



Stress at neutral axis = 0

Stress at a distance y from neutral axis

$$\sigma = (E/R)y$$

Consider a small section having area dA at a distance y from neutral axis.

Force on the layer = Stress on layer x Area of layer

$$= \sigma \times dA = (E/R)y \times dA$$

Total force on the beam section = $\int (E/R)y \times dA$

$$= (E/R) \int y dA$$

For pure bending, the force is zero.(no force on the section)

$(E/R) \int ydA = 0$

$\int ydA = 0$ (E/R cannot be 0)

ydA represents moment of area dA about neutral axis.

$\int ydA$ represents moment of entire area of the section about the neutral axis.

Moment of any area about an axis passing through its centroid, is also equal to zero.

Hence, neutral axis coincides with the centroidal axis.

Centroidal axis of a section gives the position of the neutral axis.

Moment of Resistance

Due to pure bending, layers above the N.A. are subjected to compressive stresses whereas layers below the N.A. are subjected to tensile stresses.

Forces will be acting on the layers due to these stresses.

These forces will have moment about N.A.

Total moment of these forces about the N.A. for a section is known as moment of resistance of that section (M_r).

Force on the layer at a distance y from N.A. = (E/R) y dA

Moment of this force about N.A. = Force on layer x y

= (E/R) y dA x y

Moment of Resistance…

Total moment of forces on the section of the beam (or moment of resistance) = $\int (E/R)y.ydA$ = $E/R \int y^2 \, dA = M_r$

Let M = External moment applied on the beam section.

For equilibrium the moment of resistance offered by the section (M_r) should be equal to the external bending moment (M).

$$M = E/R \int y^2 \, dA$$

Moment of Resistance…

But the expression $\int y^2\, dA$ represent moment of inertia (I)

$$M = (E/R).I \text{ or } M/I = E/R$$

But we have, $\sigma/y = E/R$

the bending equation

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Bending stresses in symmetrical sections

N.A of symmetrical section(rectangular, circular or square) lies at (d/2) from outermost layer of the section.

No stress at **N.A.** **But stresses at a point is direc**tly proportional to distance from neutral axis.

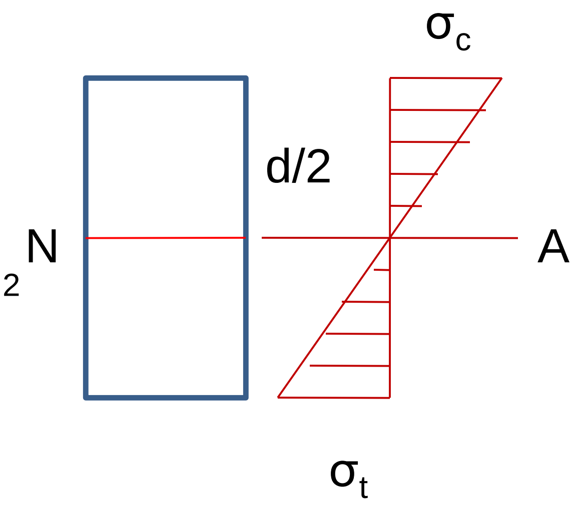
Max. stress at outermost layer

Q.Steel plate120mm wide & 20 mm thick bent in to a circular arc of 10m radius. Find

σ_{max} and corresponding BM

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

σ max.= 200 N/mm^2
M=1.6kNm



Cast iron pipe of external dia. 40 mm, internal dia. 20 mm, length 4m , supported at its ends carries 80 N at centre. Calculate max. stress induced

$$BM = \frac{WL}{4} = \frac{80 \times 4000}{4} = 8 \times 10^4 \, Nmm$$

$$I = \frac{\pi \times (D^4 - d^4)}{64} = 117809.7 mm^4$$

$$\frac{M_{\max}}{I} = \frac{\sigma_{\max}}{y_{\max}}$$

$$\sigma_{\max} = 13.58 N / mm^2$$

Section Modulus

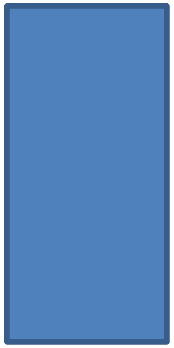
- Ratio of (M.I) of a section about N.A to distance of outer most layer from N.A

$$Z = \frac{I}{y_{\max.}} \qquad \frac{M}{I} = \frac{\sigma}{y}$$

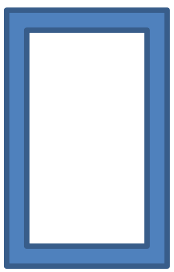
$$\frac{M}{I} = \frac{\sigma_{\max.}}{y_{\max.}} \qquad M = \sigma_{\max} \frac{I}{y_{\max.}} = \sigma_{\max} Z$$

M is the B.M. or moment of resistance offered by the section
M_{max} will get when Z is maximum
Hence section modulus represent the strength of the specimen

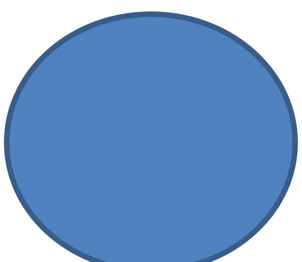
- Section modulus for various shapes of beam sections



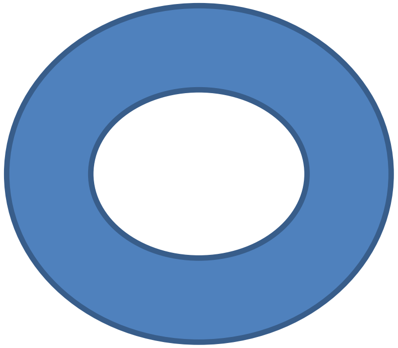
$$I = \frac{bd^3}{12}; y_{\max.} = \frac{d}{2}; z = \frac{I}{y_{\max.}} = \frac{bd^2}{6}$$



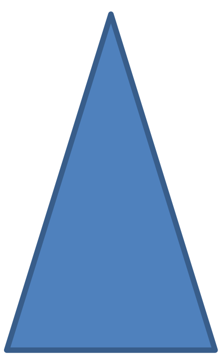
$$I = \frac{BD^3}{12} - \frac{bd^3}{12}; y_{\max.} = \frac{D}{2}; z = \frac{I}{y_{\max.}} = \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{\frac{D}{2}}$$



$$I = \frac{\pi d^4}{64}; y_{\max.} = \frac{d}{2}; z = \frac{I}{y_{\max.}} = \frac{\pi d^3}{32}$$



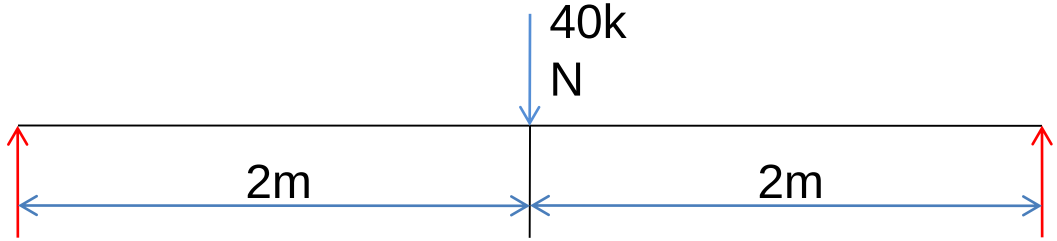
$$I = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}; y_{\text{max.}} = \frac{D}{2}; z = \frac{I}{y_{\text{max.}}} = \frac{\frac{\pi D^4}{64} - \frac{\pi d^4}{64}}{\frac{D}{2}}$$



$$I = \frac{BH^3}{36}; y_{\text{max.}} = \frac{2H}{3}; z = \frac{I}{y_{\text{max.}}} = \frac{\frac{BH^3}{36}}{\frac{2H}{3}} = \frac{BH^2}{24}$$

Q. A simply supported beam of length 4 m carries a point load of 40 kN at the centre. Find the cross section of beam assuming depth to be twice the width. The maximum bending stress in beam is not exceed 200 N/mm^2 .

A.



$$M/I = \sigma/y$$

$$M = Wl/4 = (40x4)/4$$

$$= 40x10^6 \ N.mm$$

$\sigma = 200 \text{ N/mm}^2$

$M = \sigma (I/y)$

$= \sigma . Z$

$= (\sigma (bd^2)/6$

But $d = 2b$

$\therefore Z = (b(2b)^2)/6 = (4b^3)/6$

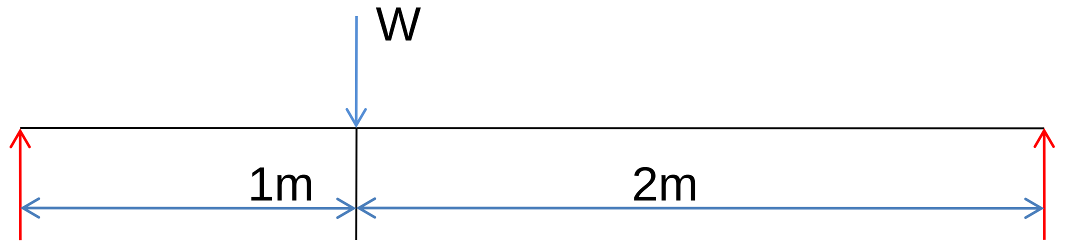
$\therefore 40 \times 10^6 = 200 \times ((4b^3)/6)$

$b = 66.94$ mm

$d = 133.89$ mm

Q. A circular pipe of external diameter 75mm and thickness 8 mm is used as a simply supported beam over a span of 3 m. Find the maximum concentrated load that can be applied at one third span if permissible stress in tube is 150 N/mm^2 .

A.



$$M = W.a.b/l = (W x 1 x 2)/3$$

$$= (2W)/3 \text{ N.m}$$

$\sigma = 150\ N/mm^2$

$M = \sigma . Z$

$$Z = \frac{\Pi}{32D}(D^4 - d^4)$$

D = 75 mm

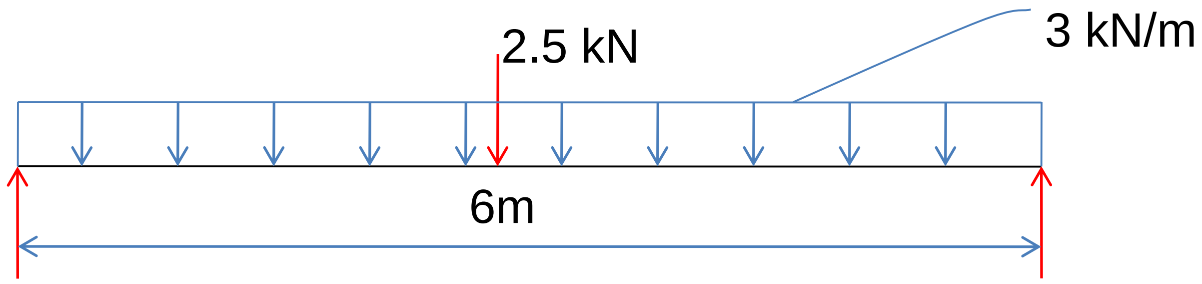
d = 75 – 2 x 8 = 59 mm

$Z = 25555.893\ mm^3$

$$\therefore \quad \frac{2W}{3} \times 10^3 = 150 \times 25555.893$$

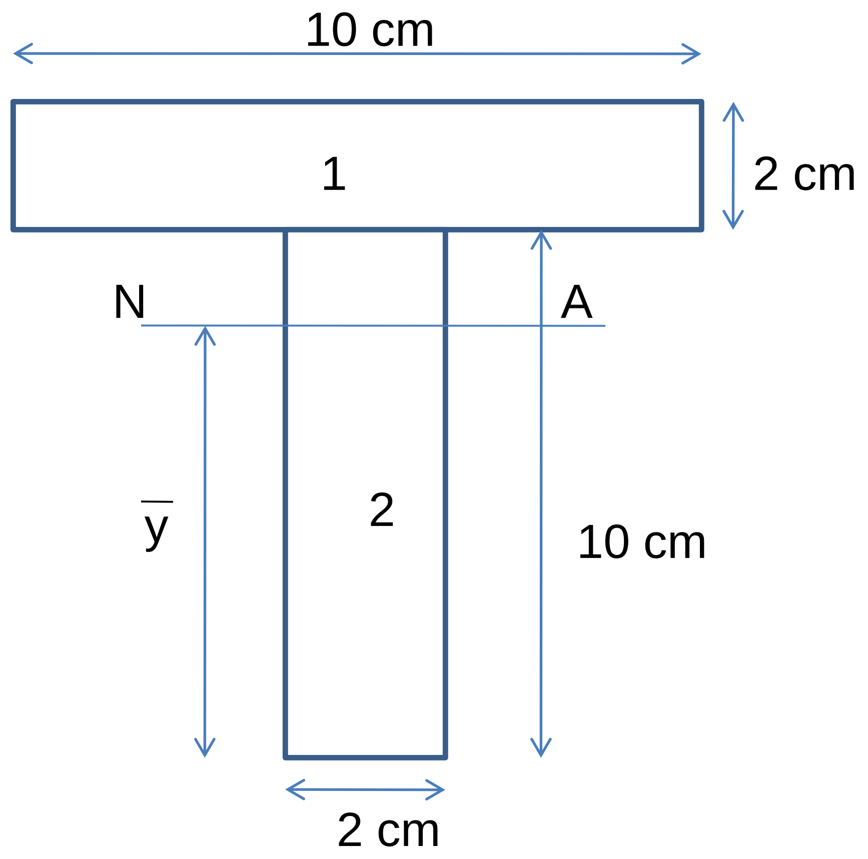
W = 5750.08 N = 5.75 kN

Q. A T section beam having flange 2 cm x 10 cm and web 10 cm x 2 cm is simply supported over a span of 6m. It carries a udl of 3kN/m run including its own weight over its span, together with a load of 2.5 kN at a mid span. Find the maximum tensile and compressive stresses occurring in the beam section.



$$M = (Wl/4) + (Wl^2/8) = (2.5 \times 6)/4 + (3 \times 6^2)/8$$

$$= 17.25 \times 10^6 \text{ N.mm}$$



To get M.I we have to locate centroid and get $\bar{y}$

No	Area	y	Ay	I_{gg}	$h = (\bar{y} - y)$	Ah^2	$I_{xx} = I_{gg} + Ah^2$
1	20	5	100	$(2 \times 10^3)/12$	3	180	346.67
2	20	11	220	$(10 \times 2^3)/12$	3	180	186.67

$\Sigma A = 40 \text{ cm}^2 \quad \Sigma Ay = 320$

$$\bar{y} = \frac{\sum Ay}{\sum A} = 8cm$$

$I_{xx} = 533.33 \text{ cm}^4$

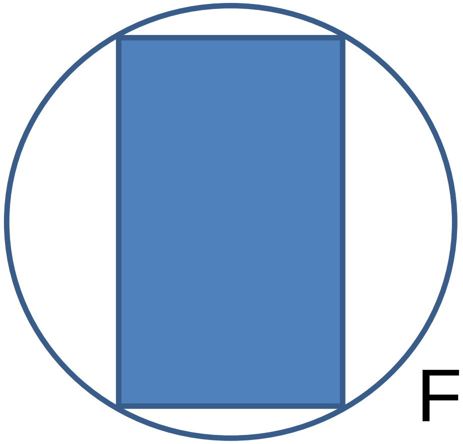
maximum tensile stress at bottom fibre, $\sigma = \frac{M}{Z} = \frac{M}{I} y$

$$\frac{17.25x10^6 \times 80}{533.33 \times 10^4} = 258.75 N / mm^2$$

maximum compressive stress at topmost fibre, $= \frac{M}{I} y$

$$\frac{17.25 \times 10^6 \times 40}{533.33 \times 10^4} = 129.375 N / mm^2$$

- Calculate the dimension of the strongest rectangular section that can be cut out from a timber log of 25 cm diameter
- b and d be the dimensions

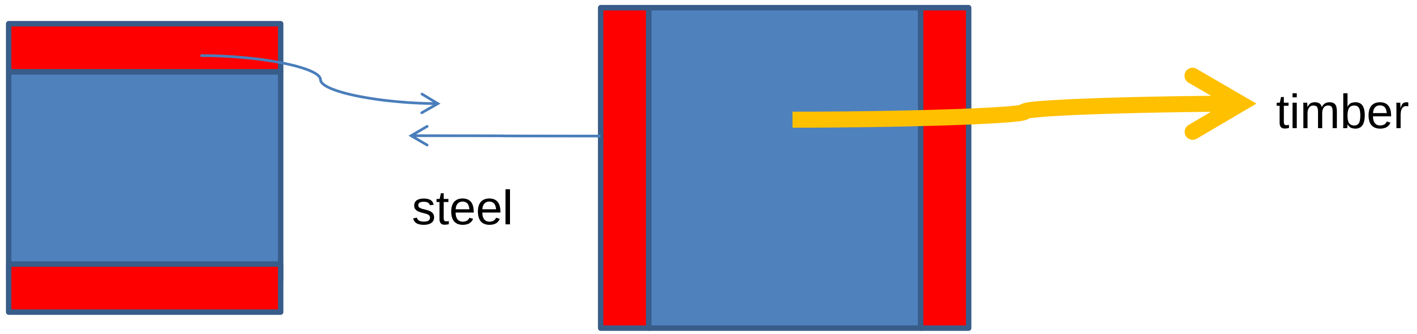


- $b^2+d^2=25^2$; $d^2=25^2 - b^2$
- Section modulus of rectangular
- section= $bd^2/6$ = $b(25^2 - b^2)/6$
- For a beam to be strong z to be max.

- $25^2-3b^2=0$ b=14.43 d=20.42 mm

Composite beam(Flitched beam)

- Consists of two different materials---eg. Timber beam reinforced with steel plates at top and bottom or on sides



- 2 materials are connected rigidly, strain same at any distance from neutral axis

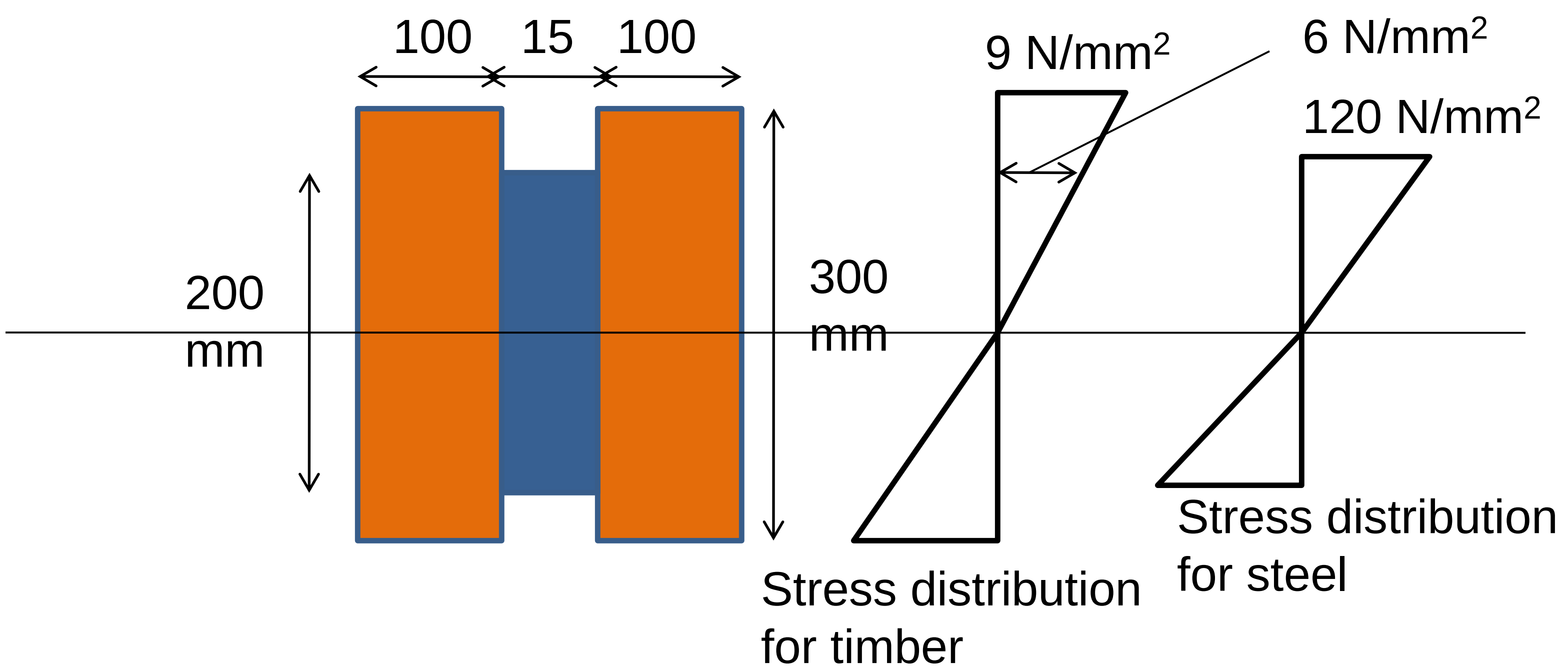
$$\frac{\sigma_1}{E1} = \frac{\sigma_2}{E_2}$$

Total resisting moment=sum of resisting moment caused by individual material

M=M1+M2

A composite beam consist of two timber joists 100 mm wide and 300 mm deep with a steel plate 200 mm deep and 15 mm thick placed symmetrically in between and clamped to them. Calculate the total moment of resistance of the section if the allowable stress in the joist is 9 N/mm^2 assume $E_s = 20E_w$

A.



In the stress diagram for timber the allowable stress is 9 N/mm^2 (maximum in joist)

at the level of steel, stress in timber = $\frac{9}{150}\times 100 = 6 N/mm^2$

We know that $\frac{\sigma_s}{E_s} = \frac{\sigma_w}{E_w}$

$$\therefore \sigma_s = \sigma_w \frac{E_s}{E_w} = 6\times 20 = 120 N/mm^2$$

This is the maximum stress in steel

$$M = M_S + M_W = \sigma_s Z_s + \sigma_w Z_w$$

$$Z_s = \frac{bd^2}{6} = \frac{15\times 200^2}{6} = 10^5 mm^3$$

$$Z_s = \frac{bd^2}{6} = \frac{200 \times 300^2}{6} = 3 \times 10^6 \, mm^3$$

$$M = M_S + M_W = \sigma_s Z_s + \sigma_w Z_w$$

$$= 120 \times 10^5 + 9 \times 3 \times 10^6 = 39 \times 10^6 \, Nmm$$