Limit state of Serviceability

- Limit State of Serviceability-
  - limit state of deflection-
  - short term and long term deflection-
  - IS code recommendations-

- limit state of cracking-
  - estimation of crack width-
  - numerical examples

Need for Limit State of Serviceability

1. Structures designed by limit state of collapse are found to be of comparatively smaller sections.
2. They must be checked for deflection and width of cracks.
3. Excessive deflection of a structure or part thereof adversely affects the appearance and efficiency of the structure, finishes or partitions.
4. Excessive cracking of concrete also seriously affects the appearance and durability of the structure.

Short- and Long-term Deflections

Short-term deflection refers to the immediate deflection after casting and application of partial or full service loads.

The following factors influence the short-term deflection

1. *magnitude and distribution of live loads,*
2. *span and type of end supports,*
3. *cross-sectional area of the members,*
4. *amount of steel reinforcement and the stress*
5. *developed in the reinforcement,*
6. *characteristic strengths of concrete and steel,*
7. *amount and extent of cracking. ,*

- Long-term deflection occurs over a long period of time largely due to shrinkage and creep of the materials.
- long-term deflection is almost two to three times of the short-term deflection.
-
- factors influencing the long-term deflection
- *humidity and temperature ranges during curing,*
- *age of concrete at the time of loading*
- *type and size of aggregates,*
- *water-cement ratio,*
- *amount of compression reinforcement,*
- *size of members etc.*

Code Provisions

Clause 35.3 of IS 456 refers to the limit state of serviceability comprising

1. Deflection in CL. 35.3.1 and
2. Cracking in CL. 35.3.2.

Control of Deflection

CL 23.2

The deflection shall generally be limited to the following:

a) The final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from the as-cast level of the supports of floors, roofs and all other horizontal members, should not normally exceed span/250.

b) The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes should not normally exceed span/350 or 20 mm whichever is less.

both the requirements are to be fulfilled

Design Procedure to Limit Deflections within acceptable limits as per IS 456-2000

1. Empirical Method

    based on Span to effective depth ratios

2. Deflection Computations

    Computing actual deflections due to short term and long term effects as per Annex C

Span to effective depth ratios *CL 23.2.1*

Concept recommended by IS 456 to be used as the guidelines to assume the initial dimensions of the members which will generally satisfy the deflection limits.

CL 23.2.1 stipulates different span to effective depth ratios

Basic values of Span to Effective Depth ratios

23.2.1 The vertical deflection limits may generally be assumed to be satisfied provided that the span to depth ratios are not greater than the values obtained as below:

a) Basic values of span to effective depth ratios for spans up to 10 m:

Cantilever	7
Simply supported	20
Continuous	26

Modification Factor (ks) for Span > 10m

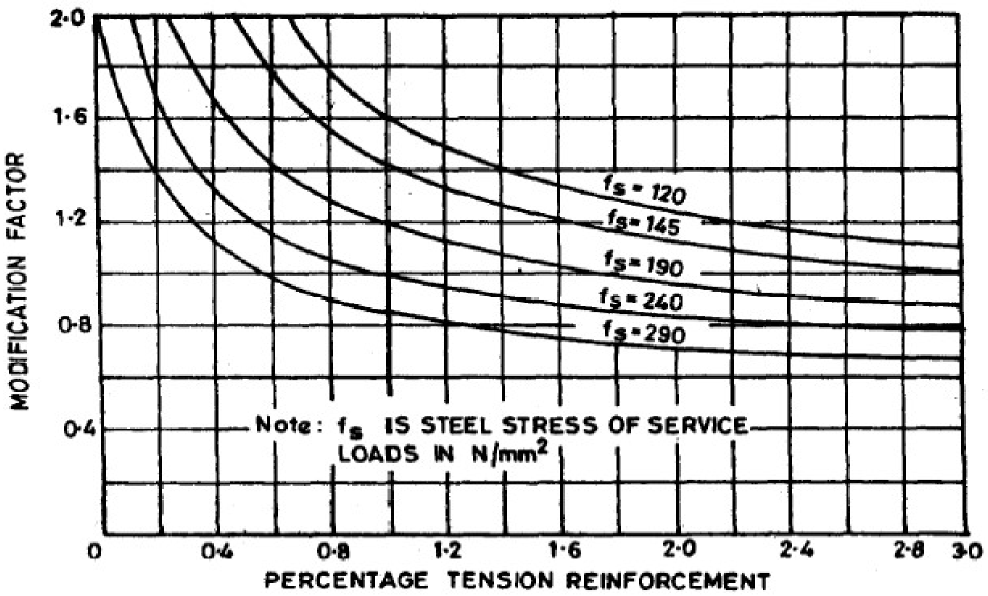
b) For spans above 10 m, the values in (a) may be multiplied by 10/span in metres, except for cantilever in which case deflection calculations should be made.

ks = 10/ Span in m

L/d = Basic Value x ks

Modification factor for Tension Reinforcement (kt)

c) **Depending on the area and the stress of steel for tension reinforcement, the values in (a) or (b) shall be modified by multiplying with the modification factor obtained as per Fig. 4.**



$$f_s = 0.58 f_y \frac{\text{Area of cross-section of steel required}}{\text{Area of cross-section of steel provided}}$$

L/d = Basic Value x ks x kt

Modification factor for Compression Reinforcement (kc)

d) Depending on the area of compression reinforcement, the value of span to depth ratio be further modified by multiplying with the modification factor obtained as per Fig. 5.

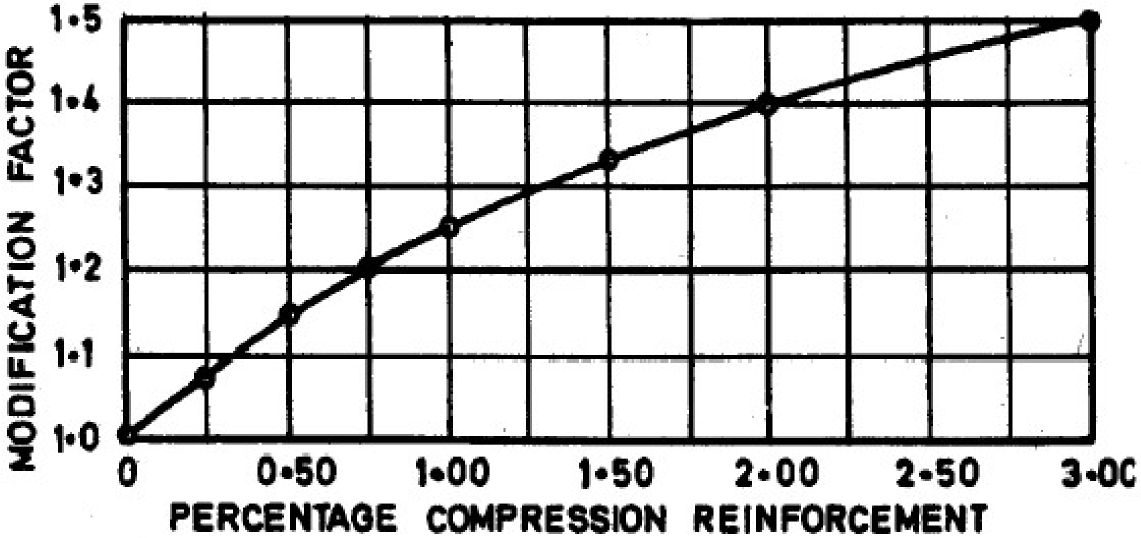


FIG. 5 MODIFICATION FACTOR FOR COMPRESSION REINFORCEMENT

L/d = Basic Value x ks x kt x kc

Reduction factor for Flanged Beams (kf)

e) For flanged beams, the values of (a) or (b) be modified as per Fig. 6 and the reinforcement percentage for use in Fig. 4 and 5 should be based on area of section equal to $b_f d$.

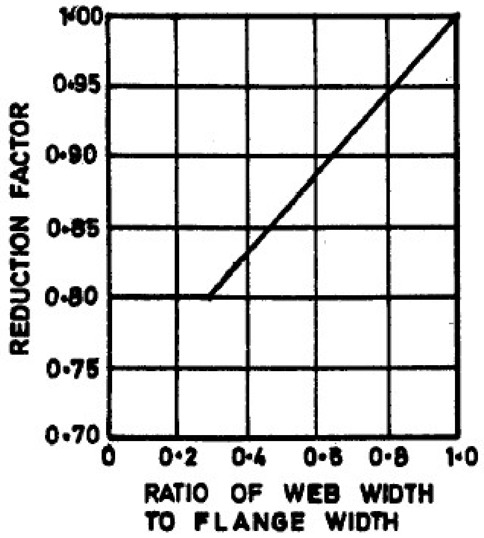


FIG. 6 REDUCTION FACTORS FOR RATIOS OF SPAN TO EFFECTIVE DEPTH FOR FLANGED BEAMS

L/d = Basic Value x ks x kt x kc xkf

Example 1

Check for the limit state of deflection using empirical method for the rectangular beam with following data.
Ast = 4- #20; Asc = 2-#16, Beam Size = 300x650 mm
Effective cover = 50mm. Fe415. Span= 12m and simply supported.

Step1

$d = 650 - 50 = 600mm$

$Ast = 1256\ mm^2$; $Asc = 402\ mm^2$

$pt = 100 \times 1256/(300 \times 600) = 0.7\%$

$pc = 100 \times 402/(300 \times 600) = 0.22\%$

<u>For Spans up to 10m :</u>

L/d (Basic Value) = 20 CL 23.2.1 (a)

Step 2: Modification factors

<u>Span > 10 mm</u>

- $ks = 10/12 = 0.833$ CL 23.2.1 (b)

<u>pt = 0.7%</u> CL 23.2.1 (c)

- $fs = 0.58\ fy = 240\ N/mm^2$
- From Fig 4; kt = 1.1

<u>pc = 0.22%</u> CL 23.2.1 (d)

- From Fig 5; kc = 1.07

Rectangular Beam

- *bw/bf =1 ; From Fig6*
- kf = 1 CL 23.2.1 (e)

Step 3: Check (L /d) ratio

Permissible Value

L/d = 20 x 0.833 x 1.1 x 1.07 x 1 = 19.6

Actual Value

L/d = 12x1000/600 = 20 > 19.6 *NOT OK*

Example 2

Check for the limit state of deflection using empirical method for the T beam with following data.

Ast = 1600 mm^2; Asc = 900 mm^2, b_w= 300mm, d=400mm, b_f= 900mm; Fe415; Span= 8m and continuous

Step1

d =400mm

pt = 100 x1600/(900x400) = 0.44%

pc = 100 x900/(900x400) = 0.25%

For Spans up to 10m :

L/d (Basic Value) = 26 CL 23.2.1 (a)

Step 2: Modification factors

Span < 10 mm

- ks = 1 CL 23.2.1 (b)

pt = 0.44% CL 23.2.1 (c)

- fs = 0.58 fy = 240 N/mm^2
- From Fig 4; kt = 1.3

pc = 0.25% CL 23.2.1 (d)

- From Fig 5; kc = 1.08

Flanged Beam

- *bw/bf =300/900 = 0.33 ; From Fig6*
- kf = 0.8 CL 23.2.1 (e)

Step 3: Check (L /d) ratio

Permissible Value

L/d = 26 x 1 x 1.3 x 1.08 x 0.8 = 29.2

Actual Value

L/d = 8x1000/400 = 20 < 29.2 *OK*

Example 3 - Deflection Computation as per Annex C

A cantilever beam of span 4m and size=300x600 mm, is subjected to a Maximum bending moment,
M =160 kNm. under service conditions due to UDL.

The beam is reinforced with 4-#20 on the tension face at an effective cover=50mm.

M20 and Fe 415 steel. Compute deflection due to short and long term effects.

Step 1: Properties of Uncracked Section

Igr (Gross Moment of Inertia) = $bD^3/12$

Igr = $300 \times 600^3/12 = 54 \times 10^8\ mm^4$

yt = D/2 = 300 mm

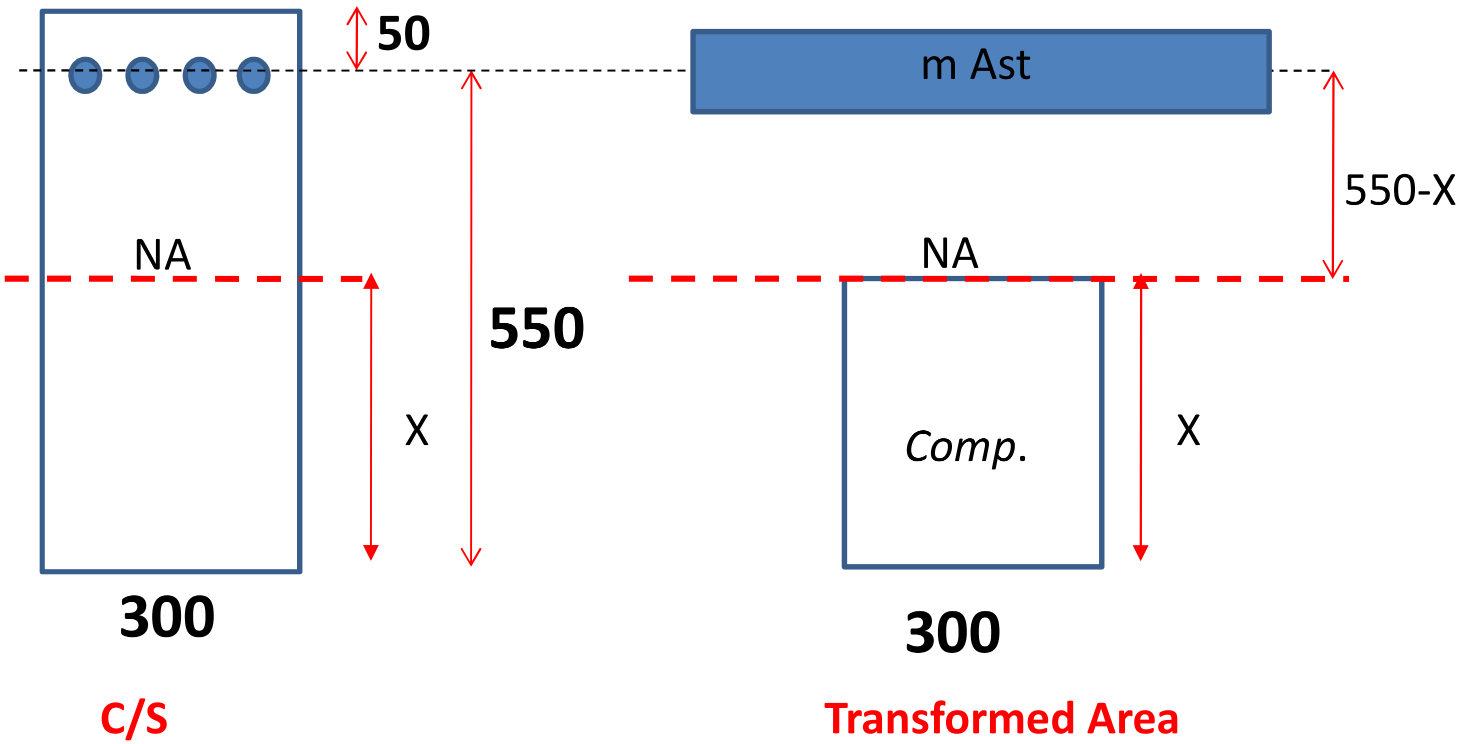
Step 2: Properties of Cracked Section

This is determined based on the area of transformed section.
Steel is replaced by an equivalent area of concrete = m Ast

Ast = 1256 mm^2 (4-#20)

Ec = 5000 $(20)^{0.5}$ = 22361 N/mm^2 *CL 6.2.3.1*

Modular ratio m = Es/Ec = 200000/22361 = 8.94



Position of NA.

Moment of Area in Compression about NA $= 300\ X^2/2$

Moment of equivalent Area in Tension about NA = m Ast (550-X)

$$300\ X^2/2 = 8.94\ x1256\ x(550-X)$$

$X = 168.9$ mm

<u>Lever arm</u>

$z = d - (X/3) = 550 - (168.9/3) = 493.7$ mm

<u>Cracked Moment of Inertia</u>

$$Ir = \{300\ x\ 168.9^3/3\} + 8.94x1256x(550-168.9)^2$$

$$=21.1\ x10^8\ mm^4$$

Cracking Moment Mr

$f_{cr} = 0.7\ (20)^{0.5} = 3.13\ N/mm^2$ Cl. 6.2.2

$M_r = f_{cr}\, I_{gr} / y_t$

$= 3.13(54)(10^8)/300 = 56.34\ kNm$

Effective Moment of Inertia Ieff C-2.1

$$Ieff = \frac{Ir}{1.2 - (Mr/M)(Z/d)(1 - X/d)(bw/b)}$$

$$Ieff = \frac{\text{Ir}}{1.2 - (56.34/160)(493.7/550)\ (1 - (168.9/550))\ (300/300)}$$

Ieff = 1.02 Ir

$I_r \leq I_{eff}$

$I_{eff} = 1.02 \times 21.1 \times 10^8 = 21.52 \times 10^8\ mm^4 < Igr$

Condition is Satisfied

Step 3: Short-term deflection (a_i)

$a_i = wL^4 / 8\, E_c I_{eff}$

$= (wL^2/2)\,(L^2/4\, E_c I_{eff})$

$= M\ L^2 / (4\, E_c I_{eff})$

$= 160 \times 10^6 \times 4000^2 / (4 \times 22361 \times 21.52 \times 10^8)$

$a_i = 13.3$ mm

Step 4: Deflection due to shrinkage C-3.1

$a_{cs} = k_3 \psi_{cs} L^2$

$k_3 = 0.5$ (cantilever)

$\psi_{cs} = k_4 \epsilon_{cs} / D$

pt = 100 x 1256/ (300 x550) = 0.76%; pc=0

(pt – pc) = 0.76

<u>For 0.25 < (pt-pc) < 1.0</u>

$k_4 = 0.72 \times 0.76 / (0.76)^{0.5} = 0.63$ **< 1.0**

$K_4 = 0.63$

$\epsilon_{cs} = 0.0003$ CL 6.2.4.1

$\psi_{cs} = 0.63 \times 0.0003 / 600 = 3.15 \times 10^{-7}$

$a_{cs} = 0.5 \times 3.15 \times 10^{-7} \times (4000)^2 = 2.52mm$

Step 5: Deflection due to Creep C-4.1

$a_{cc(perm)} = a_{icc(perm)} - a_{i(perm)}$

__Compute $a_{icc(perm)}$__

Assuming the age of concrete at loading as 28 days, CL. 6.2.5.1; $\theta = 1.6$

$E_{ce} = E_c/(1 + \theta) = 22361/(2.6) = 8600.4\ N/mm^2$

$m = E_s/E_{ce} = 200000/8600.4 = 23.25$

Position of NA

$$300\ X^2/2 = 23.25\ x1256\ x(550\text{-}X)$$

$X = 244.05\ mm$

<u>Lever arm</u>

$$z = d - (X/3) = 550 - (244.05\ /3) = 468.65\ mm$$

<u>Cracked Moment of Inertia</u>

$$Ir = \{300\ x\ 244.05^3/3\} + 23.25x1256x(550\ \text{-}244.05)^2$$

$$=41.9\ x10^8\ mm^4$$

Assume 50% of Moment as due to permanent Loads

$M = 0.5 \times 160 = 80$ kNm

Effective Moment of Inertia Ieff C-2.1

$$Ieff = \frac{\mathrm{Ir}}{1.2 - (56.34/80)(468.65/550)(1 - (244.05/550))(300/300)}$$

Ieff = 1.154Ir

$I_r \leq I_{eff}$.

$I_{eff} = 1.154 \times 41.9 \times 10^8 = 48.37 \times 10^8\ mm^4 \leq I_{gr}$

Condition is Satisfied

$$a_{icc}\ (Perm) = M\ L^2 / (4\ E_{ce}\ I_{eff})$$

$$= 80 x 10^6 x 4000^2 / (4\ x\ 8600.4 x\ 48.37 x 10^8)$$

$$= 7.69\ \text{mm}$$

Compute $a_{i(perm)}$

$$a_{i\,(Perm)} = M\ L^2 / (4\ E_c\, I_{eff})$$

$$= 80 \times 10^6 \times 4000^2 / (4 \times 22361 \times 48.37 \times 10^8)$$

$$= 2.96\text{mm}$$

$$a_{cc(perm)} = 7.69 - 2.96 = 4.73\ \text{mm}$$

Step 6: Limiting Values CL 23.2

CL 23.2.(a)

Maximum allowable deflection

= 4000/250 = 16 mm

Actual deflection = 13.3 + 2.52+ 4.73

= 20.55mm > 16 mm *NOT OK*

<u>CL 23.2.(b)</u>

Maximum allowable deflection

$= 4000/350 = 11.4$ mm < 20 mm

Actual deflection $= 2.96 + 2.52 + 4.73$

$= 10.21$mm < 11.4 mm *OK*

Limit State of Serviceability - Cracking

CL 35.3.2

<u>Maximum Crack Width</u>

< 0.3mm (Normal)

< 0.2 mm (exposed to moisture, contact with soil or ground water)

< 0.1 mm (Severe Table 3)

Control of Crack Width

1. As per General Detailing rules specified in CL 26 ; IS 456 with regard to minimum Ast, maximum spacing of rebars, minimum cover etc.

2. Crack width Computation As per Annex F

Crack width Computation As per Annex F

$$W_{cr} = \frac{3\, a_{cr}\, \varepsilon_m}{1 + \dfrac{2(a_{cr} - C_{min})}{h - x}}$$

where

a_{cr} = distance from the point considered to the surface of the nearest longitudinal bar,

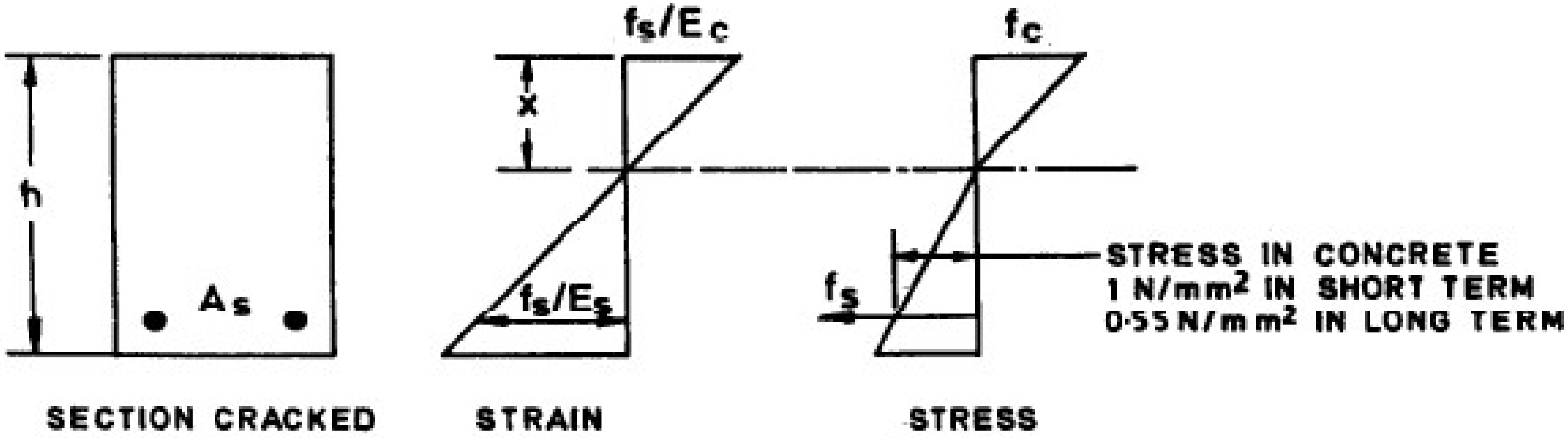
C_{min} = minimum cover to the longitudinal bar;

ε_m = average steel strain at the level considered,

h = overall depth of the member, and

x = depth of the neutral axis.

The average steel strain ε_m may be calculated on the basis of the following assumption:



f_s/E_c
f_c
x
h
A_s
f_s/E_s
f_s
STRESS IN CONCRETE
1 N/mm^2 IN SHORT TERM
0·55 N/mm^2 IN LONG TERM
SECTION CRACKED
STRAIN
STRESS

$$\varepsilon_m = \varepsilon_1 - \frac{b\,(h-x)(a-x)}{3E_s A_s (d-x)}$$

where

A_s = area of tension reinforcement,

b = width of the section at the centroid of the tension steel,

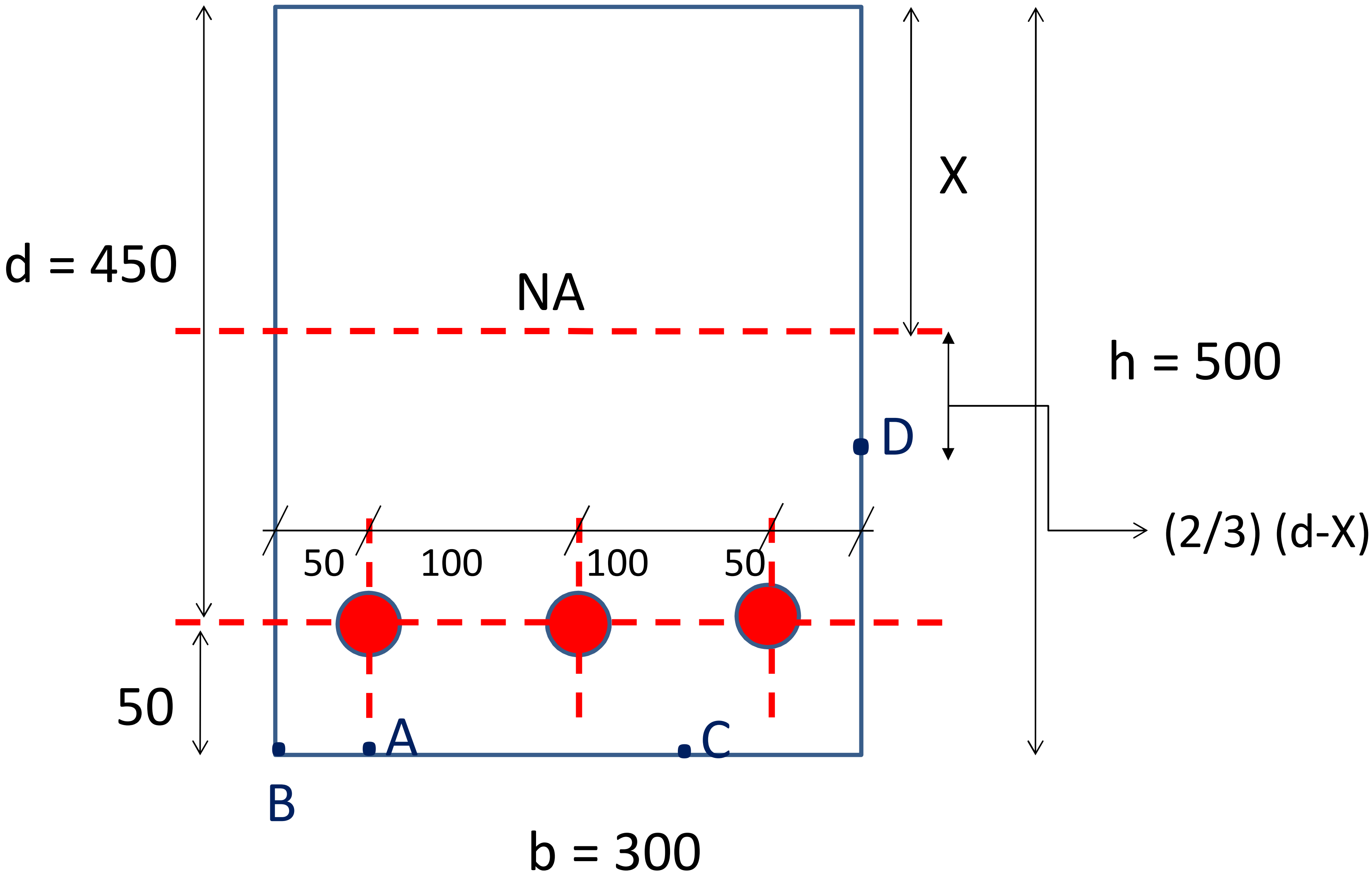
ε_1 = strain at the level considered, calculated ignoring the stiffening of the concrete in the tension zone,

a = distance from the campression face to the point at which the crack width is being calculated, and

d = effective depth.

Example

For the beam C/S shown compute crack widths at points A,B,C and D. The beam is simply supported over a span of 6m and subjected to a service load including self weight of 16 kN/m. Ast = 3- #20, M20 and Fe 415



d = 450
NA
X
h = 500
D
(2/3) (d-X)
50
100
100
50
50
A
C
B
b = 300

Step 1: Basic Parameters

$Ast = 3 - \#20 = 942\ mm^2$

$Ms = w_s\ L^2/8 = 16 \times 36/8 = 72\ kNm$

$Ec = 5000\ (20)^{0.5} = 22361\ N/mm^2$ *CL 6.2.3.1*

$Es = 200000\ N/mm^2$

$m = Es/Ec = 8.94$

Step 2: Position of NA

$300\ X^2/2 = 8.94 \times 942 \times (450-X)$

$X = 133.36$ mm

Step 3: Cracked Moment of Inertia (Ir)

$Ir = 300 \times 133.36^3/3 + 8.94 \times 942 \times (450-133.36)^2$

$Ir = 1081.52 \times 10^6\ mm^4$

Step 4: Strain in Concrete at the level of steel

ϵ_{cs} = fcs/ Ec

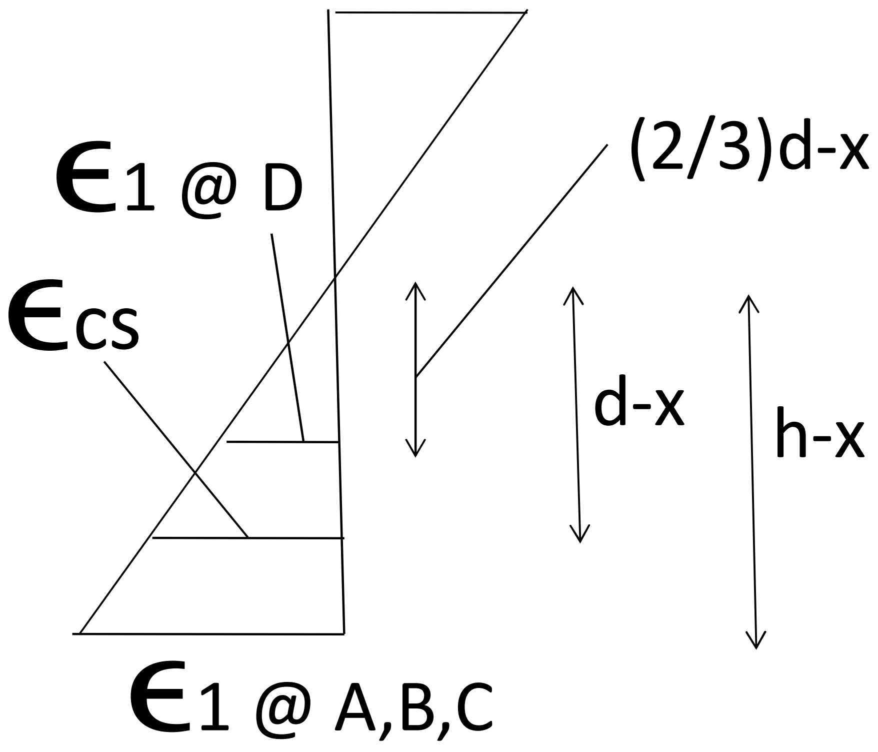
fcs = Ms(d-x)/ Ir (M/I = f/y; f = M x y /I)

$= 72 \times 10^6 (450-133.36) / 1081.52 \times 10^6$

$= 21.08 \text{ N/mm}^2$

$\epsilon_{cs} = 21.08 / 22361 = 9.43 \times 10^{-4}$

Step 5: Strain ϵ_1 at A,B,C and D



At A,B,C

$$\epsilon_1 = \epsilon_{cs}\,(h-X)/(d-X)$$

$$= 9.43\times10^{-4}\,(500-133.36)/(450-133.36)$$

$$= 1.09\times10^{-3}$$

At D

$$\epsilon_1 = \epsilon_{cs}\,(2/3)(d-X)/(d-X)$$

$$= 9.43\times10^{-4}\,(2/3)$$

$$= 6.28\times10^{-4}$$

Step 6: ϵ_m at A,B,C,D

$$\epsilon_m = \epsilon_1 - \{b(h-X)(a-X) / (3\ E_s\ A_s(d-X)\}$$

$$b(h-X) / (3\ E_s\ A_s\ (d-X) =$$

$$300(500-133.36) / (3 \times 200000 \times 942 \times (450-133.36)$$

$$= 6.15 \times 10^{-7}$$

$$\epsilon_m = \epsilon_1 - 6.15 \times 10^{-7}\,(a-X)$$

$$\epsilon_m = \epsilon_1 - 6.15\times10^{-7}(a - X)$$

At A,B,C

$a = h = 500mm$, $\epsilon_1 = 1.09\times10^{-3}$

$$\epsilon_m = \epsilon_1 - 6.15\times10^{-7}(a - X) = 8.64\times10^{-4}$$

At D

$a = X + (2/3)(d - X) = 344.45$ mm, $\epsilon_1 = 6.28\times10^{-4}$

$$\epsilon_m = \epsilon_1 - 6.15\times10^{-7}(a - X) = 4.98\times10^{-4}$$

Step 7: Crack width w_{cr}

At A

$a_{cr} = 50 - 10 = 40mm$

$C_{min} = 40mm$; $\epsilon_m = 8.64x10^{-4}$

$w_{cr} = 3 \times 40 \times 8.64x10^{-4} = 0.103mm$

At B

$a_{cr} = (50^2 + 50^2)^{0.5} - 10 = 60.7mm$

$C_{min} = 40mm$; $\epsilon_m = 8.64x10^{-4}$

w_{cr}

$= 3 \, x60.71x8.64x10^{-4} / (1 + 2(60.71 - 40)/(500 - 133.36)) =$

$0.141mm$

At C

$a_{cr} = (50^2 + 50^2)^{0.5} - 10 = 60.7mm$

$C_{min} = 40mm$

$w_{cr} = 0.141$ mm

At D

$a_{cr} = (50^2 + ((450-133.36)/3)^2)^{0.5} - 10$

$= 106.8mm$

$C_{min} = 40mm$; $\epsilon_m = 4.98x10^{-4}$

$w_{cr} = 0.117$ mm

DEFLECTIONS OF BEAMS

Deflections

- **Deflection of structure or part thereof shall not adversely affect the appearance or efficiency of structure or finishes or partitions.**
- **Deflection shall generally be limited to the following:**

*i. **Final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from as-cast level of supports of floors, roofs and all other horizontal members should not normally exceed span/250.***

*ii. **Deflection including effects of temperature, creep and shrinkage occurring after erection of partitions and application of finishes should not normally exceed (span/350) or 20mm whichever is less.***

- **Factors influencing limits on deflection in flexural members**
  - ***Aesthetic/psychological discomfort***
  - ***Crack width limitation***
  - ***Effect on attached structural and non structural elements***
  - ***Ponding in (roof) slabs***

- **Limits on deflections of flexural members**

i. ***Final deflection due to all loads including temperature effect, long term effects of creep and shrinkage after construction of structural members and before the construction of partitions and finishes should be less than*** $\boxed{\frac{Span}{250}}$

ii. ***Deflections due to various loads acting on the structural members including temperature effect, long-term effects of creep and shrinkage that occur after the construction of partitions and finishes should be less than*** $\boxed{\frac{Span}{350}}$ ***or 20 mm***

Short-Term Deflections

- Short-term deflections due to the applied service loads are generally based on the assumptions of linear elastic behavior, and for this purpose, reinforced concrete is treated as a homogenous material.

$$\Delta = k_w \frac{WL^3}{EI}$$

$$= k_m \frac{ML^2}{EI}$$

Where

- W = total load on the span

  M = maximum moment

  K_w and K_m – constants depend on the load distribution conditions of end restraint and variation in the flexural rigidity EI.

- For the purpose of calculating short–term deflections in reinforced concrete flexural members elastic theory may be used of.
- Flexural rigidity is to be considered in the calculation.

Short-Term Deflections

- Modulus of elasticity of concrete depends on:
  - Concrete quality,
  - Age
  - Stress level and
  - Rate or duration of applied load.
- Short-term loading upto service load levels, IS:456-2000 specifies the modulus of elasticity as $E_c=5000\sqrt{f_{ck}}$.
- Second moment of area, I, to be considered in the deflection calculations is influenced by
  - Percentage reinforcement
  - Extent of flexural cracking,
- which in turn depends on the applied bending moment and the modulus of rupture, f_{cr}, of concrete.

- **Flexural rigidity, EI is obtainable as the slope(secant modulus) of the moment-curvature relationship.**

  i. **EI_T – Based on uncracked transformed section**

  ii. **EI_{gr} – Based on gross uncracked transformed section ignoring steel**

  iii. **EI_{eff} – Based on effective section**

  iv. **EI_{cr} – Based on "cracked transformed section"**

✓ **EI_T - True EI for $M < M_{cr}$ (EI_T - constant)**

✓ **EI_{eff} - True EI for $M > M_{cr}$ (EI_{eff} – depends on the load level)**

$$EI_T > EI_{gr} > EI_{eff} > EI_{cr}$$

- **IS: 456-2000 specifies,**

$$I_{eff} = \frac{I_{cr}}{1.2 - \left(\frac{M_{cr}}{M}\right)\left(\frac{z}{d}\right)\left(1 - \frac{x}{d}\right)\left(\frac{b_w}{b}\right)}$$

- **$I_{cr} \leq I_{eff} \leq I_{gr}$**
- **I_{cr} = Moment of inertia of the cracked section**
- **M_{cr} = Cracking moment**

$$M_{cr} = f_{cr} \frac{I_{gr}}{y_t}$$

- **f_{cr} = Modulus of rupture of concrete**
- **I_{gr} = Moment of inertia of the gross section about the centroidal axis, neglecting the reinforcement.**
- **y_t = Distance from centroidal axis of gross section, neglecting reinforcement to extreme fiber in tension.**
- **M = Maximum moment under service load**
- **Z = lever arm**
- **x = Depth of neutral axis**
- **D = Effective depth**
- **b_w= Breadth of web and,**
- **b = Breadth of compression face**

- ***For continuous beams, deflections shall be calculated using the values of I_{cr}, I_{gr} and M_{cr}, modified by the following equation:***

$$X_e = k_1 \frac{(X_1 + X_2)}{2} + (1 - k_1) X_0$$

k_1 = coefficient

X= value of I_{cr}, I_{gr}, or M_{cr} as appropriate

X_e= modified value of X

X_1 and X_2= values of "X" at the supports 1 and 2

X_o= value of "X" at mid-span.

Values of k_1

k_2	0.5 or less	1.4	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3
k_1	0	1.0	0.03	0.08	0.16	0.30	0.50	0.73	0.91	0.97

M_1, M_2 = support moments

M_{F1}, M_{F2} = fixed end moment

$$k_2 = \left(\frac{(M_1 + M_2)}{M_{F1} + M_{F2}} \right)$$

Deflections due to shrinkage

- **Deflections due to shrinkage Δ_{cs} may be computed from the equation:**

$$\Delta_{cs} = k_3 \Psi_{cs} l^2$$

k_3 = constant depending upon the support conditions

= 0.5, for cantilever

= 0.125, for simply supported members

= 0.086, for members continuous at one end

= 0.063, for fully continues members

Ψ_{cs}= shrinkage curvature $= \frac{K_4 \varepsilon_{cs}}{D}$

$$K_4 = 0.72 \frac{(P_t - P_c)}{\sqrt{P_t}} \leq 1.0$$

$$0.25 \leq (P_t - P_c) \leq 1.0$$

$$= 0.65 \frac{(P_t - P_c)}{\sqrt{P_t}} \leq 1.0 \quad \textbf{for} \quad (P_t - P_c) \geq 1.0$$

$$P_t = \frac{100 A_{st}}{bd}; P_c = \frac{100 A_{sc}}{bd}$$

D = total depth of the section

l= length of the span.

Deflections due to creep

- **Creep deflections due to permanent loads is given by,**

$$\Delta_{cc(perm)} = \Delta_{i,cc(perm)} - \Delta_{i,(perm)}$$

where,

- **$\Delta_{i,cc(perm)}$= initial plus creep deflections due to permanent loads obtained using an elastic analysis with effective modulus of elasticity.**
- **$\Delta_{i,(perm)}$= Short-term deflections due to permanent load using E_c=5000$\sqrt{f_{ck}}$**
- **Effective modulus of elasticity is,** $E_{ce} = \dfrac{E_C}{[1+\theta]}$

Age at loading	Creep coefficient, θ
7 days	2.2
28 days	1.6
1 year	1.1

Control of Deflections

- ***For control of deflection two methods are usually described in codes of practice***

I. ***As per the <u>empirical method</u>, the span-to-effective depth ratios of members should not be more than those specified in the codes***

II. ***In <u>theoretical method</u>, calculating the actual deflection and checking it with the allowable deflection in the codes of practice***

Control of Deflections

- ***Empirical procedure for control of deflection is to control the span-to-effective depth ratio.***
- ***Deflection of beams or slabs depend on***

1. ***Span-to-effective depth ratio***
2. ***Type of support***
3. ***Percentage of tension reinforcement or the stress level at service loads if more than the necessary steel is provided at the section.***
4. ***Percentage of compression reinforcement***
5. ***Type of beam (whether it is flanged or rectangular)***

- ***Vertical deflection limits may generally be assumed to be satisfied provided that the span-to-depth ratios are not greater than the values obtained***

Basic Span-to-depth Ratios

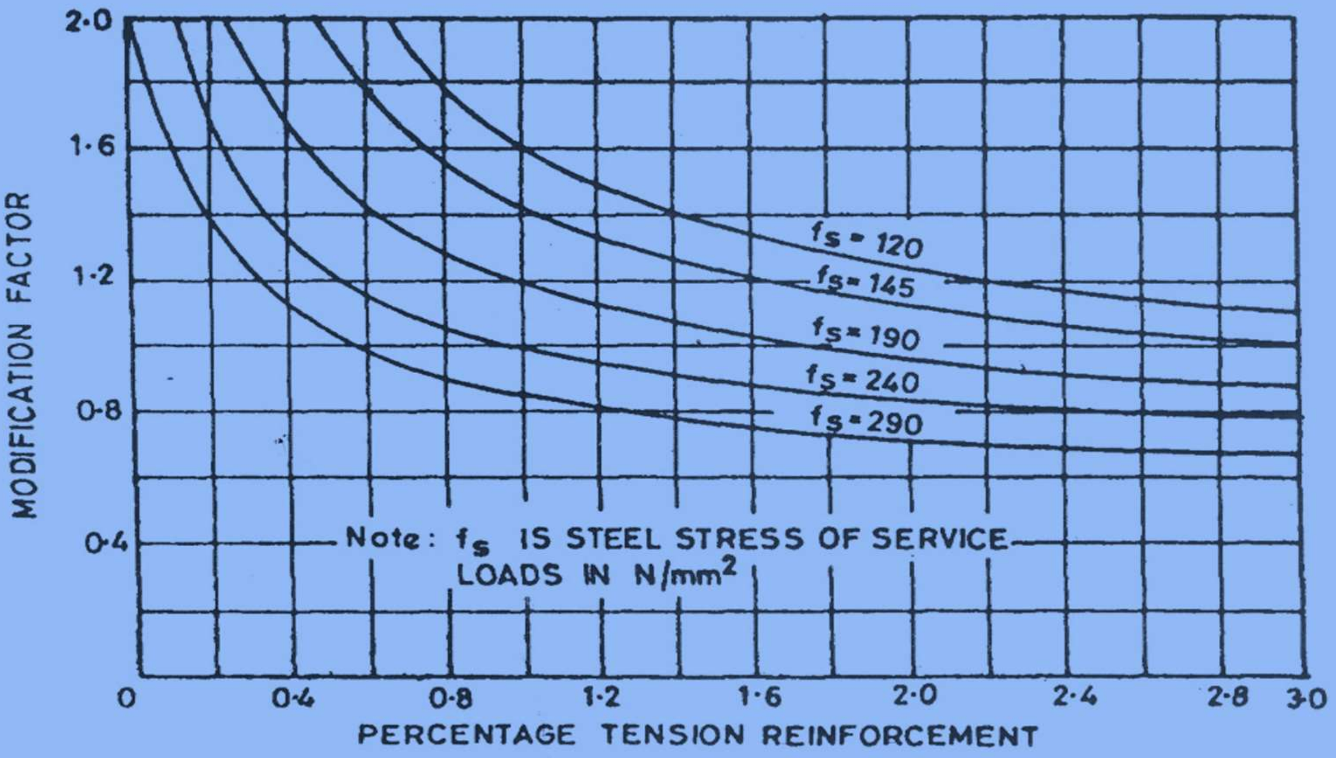
a. ***Basic values of span-to-effective depth ratios for spans up to or less than 10m are***

1.	***Cantilever***	***7***
2.	***Simply supported***	***20***
3.	***Fixed or Continuous***	***26***

b. ***For spans above 10m, <u>values in (a)</u> may be multiplied by (10/span in meters), except for cantilever.***

– ***For cantilever beams, the actual deflections should be calculated and the requirement for limit state of deflection be checked.***

1. ***Depending on the area and the stress in steel reinforcement in tension, the values in (a) or (b) shall be modified by multiplying with the modification factor obtained as per Fig. 4.***



$$f_s = 0.58\, f_y \frac{\text{Area of cross-section of steel required}}{\text{Area of cross-section of steel provided}}$$

FIG. 4 MODIFICATION FACTOR FOR TENSION REINFORCEMENT

Factor F_1 can be calculated

$$F_1 = \frac{1}{\left(0.225 + 0.00322\, f_s + 0.625 \log_{10} p_t\right)} \le 2.0$$

2. ***Depending on the area of compression reinforcement, the value of span-to-depth ratio is further modified by multiplying with the modification factor obtained as per Fig. 5.***

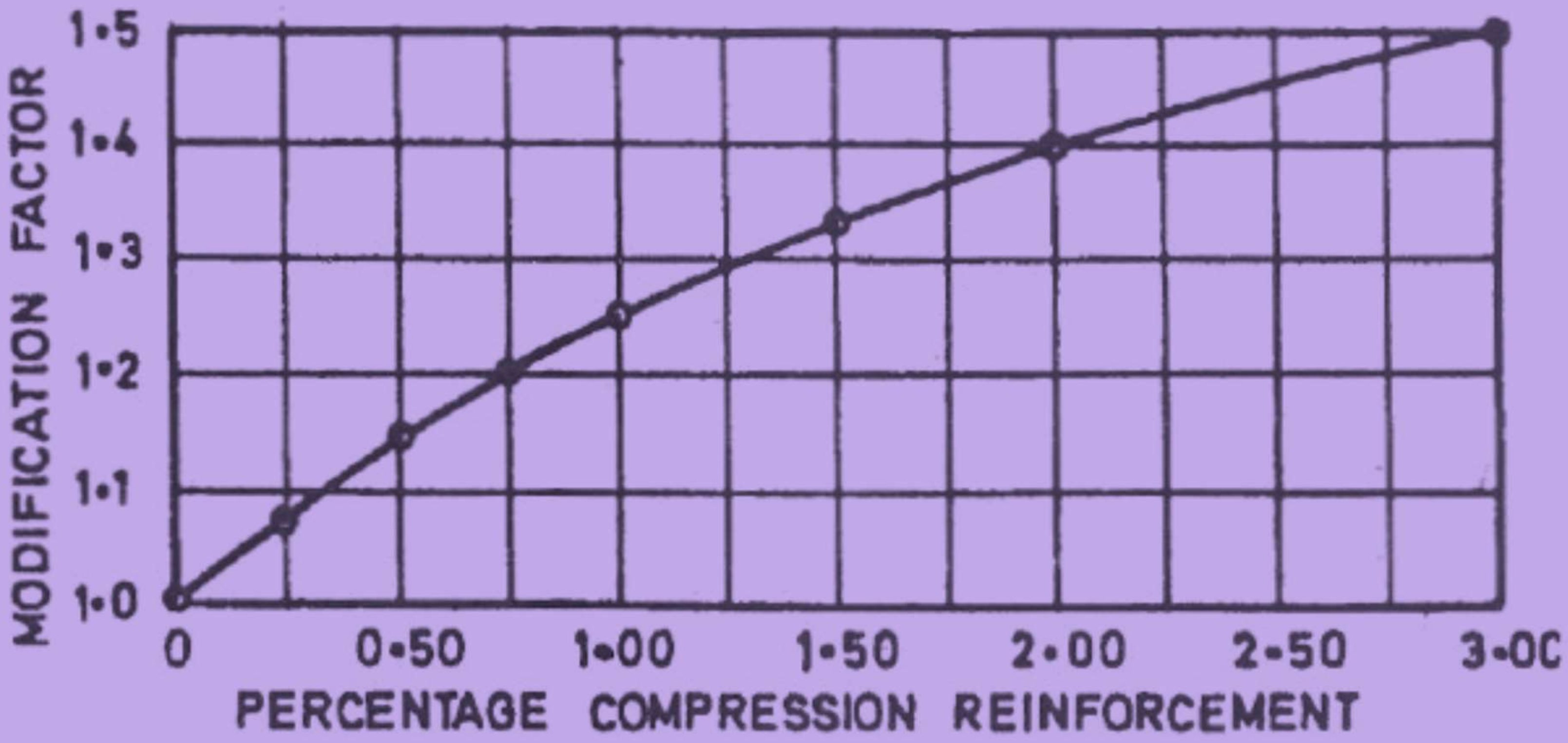


FIG. 5 MODIFICATION FACTOR FOR COMPRESSION REINFORCEMENT

Factor F_2 can be calculated

$$F_2 = \frac{1.6 p_c}{\left(p_c + 0.275\right)} \leq 1.5$$

3. For flanged beam, values of (a) or (b) be modified as per Fig. 6 and the reinforcement percentage for use in Fig.4 and 5 should be based on area of section equal to $b_f d$.

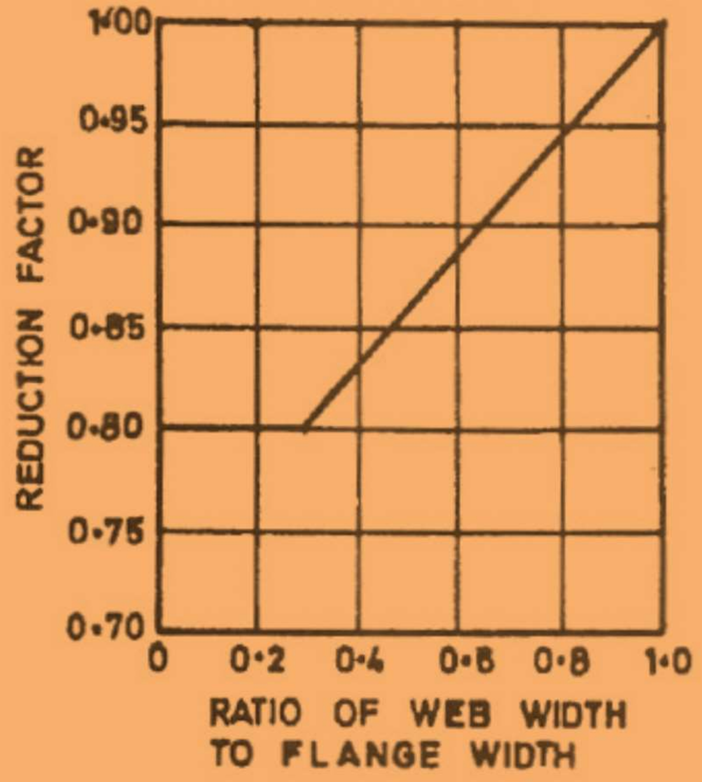


FIG. 6 REDUCTION FACTORS FOR RATIOS OF SPAN TO EFFECTIVE DEPTH FOR FLANGED BEAMS

Factor F_3 can be calculated

$$F_3 = 0.8 + \frac{2}{7}\left(\frac{b_w}{b_f} - 0.3\right) \leq 0.8$$

Final Span-to-Effective Depth Ratio

- **The final allowable span-to-effective depth ratio**

$$\frac{L}{d} = [Basic \quad Ratio\,](F_1)(F_2)(F_3)$$

- **The ratio (L/d) obtained should be as follows**

$$\frac{L}{d}\,(obtained\,) \leq \left[\frac{L}{d}\right]_{Basic} (F_1)(F_2)(F_3)$$

Slenderness Limits for Beam to Ensure Lateral Stability

- **A simply supported or continuous beam shall be so proportioned that clear distance between the lateral restraints does not exceed 60b or $\frac{250b^2}{d}$ whichever is less.**

Where

d = effective depth of the beam and

b = breadth of compression face midway between the lateral restraints.

- **For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed 25b or $\frac{100b^2}{d}$**
- **whichever is less.**

1. **A simply supported beam of cross section of width 200mm and overall depth 400mm is provided with 3- 16 mm diameter HYSD bars in tension. Cover to the reinforcement is 40mm. The span of the beam is 5.0m. The beam is subjected to a uniformly distributed dead load of 10 kN/m and a live load of 15 kN/m. Half of the imposed load is permanent. Calculate the total long-term deflection at the mid-span. f_{ck}=40 N/mm^2, f_y=415 N/mm^2, θ=2.5=creep coefficient, ε_{cs}=0.0003**

2. **A simply supported beam rectangular in cross section, 450mmX750mm, spanning 10m is subjected to a dead load of 24 kN/m and an imposed load of 34.5 KN/m. The characteristic concrete and steel strengths are f_{ck}=40 N/mm^2 and f_y=460 N/mm^2 respectively, E_s= 200 kN/m^2 and E_c=5000$\sqrt{f_{ck}}$. (A_{st}= (3)- 40mm dia bars).**

   i. **Determine the mid-span service-load concrete strains at the level of the tension reinforcements at the tension face (i.e. the soffit) of the beam, and at 250 mm below the neutral axis.**

   ii. **If, due to creep of concrete, the value of E_c becomes half of the short-term value, calculate the strain due to the long-term service loads.**

1. **A beam 200mm wide and 400mm overall depth is reinforced with 2 nos of 20mm diameter bars is acted on by a load, part of which is permanent. The bending moment due to the total loading is50 kN-m and the bending moment, M_p due to the permanent load is 36 kN-m. Assuming the section as a partially cracked section, determine, the long-term curvature of the beam under permanent load, if f_{ck}=0.55 N/mm^2 appropriate to long-term loading, the instantaneous curvature under the total load and the permanent loads, if f_{ck}=1.0 N/mm^2 for short-term loading and the difference between the instantaneous curvature under the total and permanent load. Given that $E_c=5000\sqrt{f_{ck}}$, E_s=200 kN/m^2, θ=2.5, f_{ck}=40 N/mm^2 and f_y=460 N/mm^2.**